



De Mathematics Competitions

2nd Annual

DMC 12 B

Friday, October 23, 2026



INSTRUCTIONS

1. DO NOT TURN TO THE NEXT PAGE UNTIL YOUR COMPETITION MANAGER TELLS YOU TO BEGIN.
2. This is a 25-question multiple-choice competition. For each question, only one answer choice is correct.
3. Mark your answer to each problem on the answer sheet with a #2 pencil. Check blackened answers for accuracy and erase errors completely. Only answers that are properly marked on the answer sheet will be scored.
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. Only blank scratch paper, rulers, compasses, and erasers are allowed as aids. No calculators, smartwatches, phones, or computing devices are allowed. No problems on the competition will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the competition, your competition manager will ask you to record your name on the answer sheet.
8. You will have 75 minutes to complete the competition once your competition manager tells you to begin.
9. When you finish with the competition, sign your name in the space provided on the answer sheet and complete the demographic information questions on the back of the answer sheet.

The problems and solutions for this DMC 12 were prepared
by the DMC Committee under the direction of:

Taiki Aiba and Ryan Shin

The DMC Committee reserves the right to disqualify scores from a school if it determines that the rules or the required security procedures were not followed.

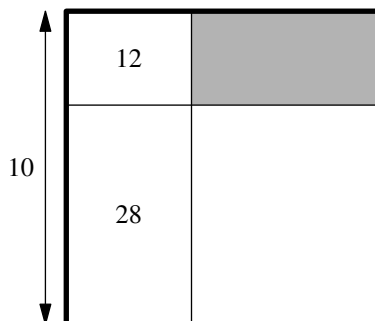
The publication, reproduction or communication of the problems or solutions of this competition during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via phone, email, or digital media of any type during this period is a violation of the competition rules.

Students who score well on this DMC 12 will be invited to take the De Invitational Mathematics Examination (DIME), which is already out and is due on Friday, January 29, 2027. More details about the DIME can be found on our site.

1. Which of the following is always greater than 0 when x and y are real numbers less than 0?

(A) $\frac{2x+y}{x^2}$ (B) $x-2y$ (C) $\frac{y}{x-y^2}$ (D) x^2+y (E) $xy+x$

2. A square with side length 10 is split into four rectangles by two lines parallel to the sides of the square. Two of those rectangles have areas 12 and 28, as shown below. What is the area of the shaded rectangle?

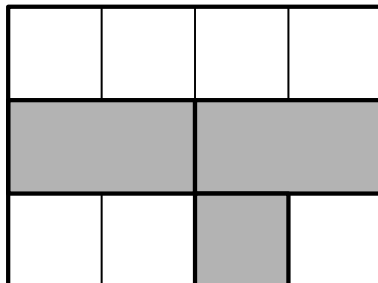


- (A) 12 (B) 15 (C) 18 (D) 24 (E) 30
3. What is the least possible value of

$$|n-2|! + |n-0|! + |n-2|! + |n-6|!$$

over all integers n ?

- (A) 9 (B) 14 (C) 28 (D) 49 (E) 123
4. Three tiles are placed on a 3×4 grid, as shown below in gray. In how many ways can the remaining squares be covered with 1×1 and 1×2 tiles so that no tiles overlap?



- (A) 8 (B) 9 (C) 10 (D) 12 (E) 15

5. A sequence of real numbers is defined recursively by $a_1 = 1$, $a_2 = 2$, and $a_n \cdot a_{n-1} = a_{n-2}$ for all $n \geq 3$. How many positive integers divide the positive integer a_{12} ?

(A) 65 (B) 72 (C) 78 (D) 90 (E) 94

6. Let $P(a, b)$ be a point in the xy -plane above the x -axis and to the right of the y -axis such that when P is reflected over the line $y = x$, its image lies on the line $y = 4x$, and when P is reflected over the x -axis, its image lies on the line $y = -\frac{x}{5} - 1$. What is $a + b$?

(A) 10 (B) 15 (C) 20 (D) 25 (E) 30

7. A triangle has side lengths 4, 5, and 7. For each vertex, Ken chooses to either draw a circle centered there with positive integer radius, or do nothing. No two circles may overlap, though they may be tangent, and no circle may contain any vertex of the triangle other than its center strictly inside it. What is the greatest possible sum of the areas of the circles?

(A) 25π (B) 26π (C) 29π (D) 34π (E) 37π

8. Armaan randomly chooses four real numbers from the interval $[-2026, 2026]$ and labels them $a_1 < a_2 < a_3 < a_4$. What is the probability that at least half of the products

$$a_1a_2, a_1a_3, a_1a_4, a_2a_3, a_2a_4, a_3a_4$$

are positive?

(A) $\frac{3}{8}$ (B) $\frac{1}{2}$ (C) $\frac{9}{16}$ (D) $\frac{5}{8}$ (E) $\frac{3}{4}$

9. Let a and b be real numbers such that $a > b > 1$ and

$$\log_a(3b) = \log_b(a^{\log_b a}) = 9.$$

What is $\log_3(ab)$?

(A) $\frac{2}{13}$ (B) $\frac{5}{26}$ (C) $\frac{4}{11}$ (D) $\frac{2}{3}$ (E) $\frac{9}{13}$

10. What is the greatest integer k such that 2^k divides n^n for some positive integer $n \leq 2026$?

(A) 2026 (B) 10,080 (C) 10,240 (D) 13,824 (E) 14,336

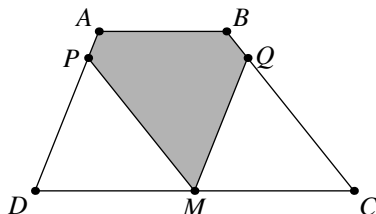
11. For how many positive integers $n \leq 100$ is

$$|1^n - n| + |2^n - 2n| + |4^n - 4n|$$

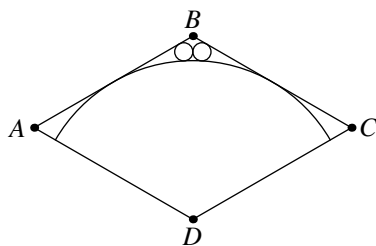
one less than a multiple of 5?

(A) 25 (B) 26 (C) 30 (D) 33 (E) 50

12. Trapezoid $ABCD$ has $\overline{AB} = 2$, $\overline{CD} = 5$, and $\overline{AB} \parallel \overline{CD}$. Point M is the midpoint of $\overline{CD}$, and points P and Q lie on $\overline{AD}$ and $\overline{BC}$, respectively, so that $\overline{MP} \parallel \overline{BC}$ and $\overline{MQ} \parallel \overline{AD}$, as shown below. What fraction of the trapezoid is shaded?



- (A) $\frac{1}{4}$ (B) $\frac{25}{84}$ (C) $\frac{1}{3}$ (D) $\frac{5}{14}$ (E) $\frac{17}{42}$
13. How many positive integer divisors does $2^{29} - 2^{14} - 1$ have?
- (A) 48 (B) 60 (C) 64 (D) 72 (E) 80
14. Rhombus $ABCD$ has side length $2\sqrt{3}$ and $\angle ABC = 120^\circ$. A 120° sector centered at D lies inside $ABCD$ whose arc is tangent to $\overline{AB}$ and $\overline{BC}$. Two circles of radius r lie inside $ABCD$ and outside the sector, are externally tangent to each other, and are each tangent to the arc of the sector. One of the circles is tangent to $\overline{AB}$, and the other is tangent to $\overline{BC}$. What is r ?



- (A) $7 - 4\sqrt{3}$ (B) $\frac{1}{6}$ (C) $3 - 2\sqrt{2}$ (D) $\frac{1}{4}$ (E) $2 - \sqrt{3}$
15. For each positive integer m , let $f(m)$ be the result when the first digit of m is moved to the end. For example, $f(123) = 231$ and $f(2026) = 0262 = 262$. What is the sum of the digits of the greatest four-digit positive integer n for which $n + f(n)$ is a perfect square?
- (A) 17 (B) 23 (C) 25 (D) 27 (E) 29
16. Let $\mathcal{P}$ be a parabola with focus F and vertex V , where $FV = 1$. Point A lies on $\mathcal{P}$, and ω is the circle centered at A that passes through F . The tangent segment from V to ω has length 5. What is the area of ω ?
- (A) 196π (B) 200π (C) 210π (D) 221π (E) 225π

17. There are four creatures in a line: a ghost, a goblin, a scarecrow, and a zombie, where exactly two of the four creatures visited the haunted house. All creatures always tell the truth, and they speak as follows:

- The ghost: “There is a creature immediately to my left. If that creature visited the haunted house, then I did not visit the haunted house.”
- The goblin: “I did not visit the haunted house.”
- The scarecrow: “If the ghost did not visit the haunted house, then the goblin must have visited the haunted house.”
- The zombie: “There is a creature immediately to my right. If that creature did not visit the haunted house, then I also did not visit the haunted house.”

In how many different orders could the creatures be standing?

- (A) 6 (B) 8 (C) 9 (D) 10 (E) 12

18. Let x , y , and z be real numbers satisfying the system of equations

$$\begin{aligned}x + y + z &= x^2 + y^2 + 2z^2, \\x + y &= \frac{2}{5}.\end{aligned}$$

The square of the difference between the greatest and least possible values of $\sqrt{y^2 + z^2}$ can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m + n$?

- (A) 141 (B) 142 (C) 143 (D) 144 (E) 145

19. In right $\triangle ABC$ with $AB < AC$ and a right angle at A , a circle is tangent to sides $\overline{AB}$ and $\overline{AC}$ at D and E , respectively, and internally tangent to the circumcircle of $\triangle ABC$ at F . Let line EF intersect the circumcircle of $\triangle ABC$ at $P \neq F$. Suppose $BP = 20$ and E is the midpoint of $\overline{AC}$. What is the area of $\triangle DEF$?

- (A) 60 (B) $45\sqrt{2}$ (C) 80 (D) $60\sqrt{2}$ (E) $50\sqrt{3}$

20. For a real number t , let $z = \cos(\pi t) + i \sin(\pi t)$. What is the sum of all t with $0 \leq t < 2$ for which z^4 , z^7 , and $\frac{1+i\sqrt{3}}{2}$ are the distinct vertices of a right triangle in the complex plane?

- (A) $\frac{26}{3}$ (B) $\frac{34}{3}$ (C) $\frac{38}{3}$ (D) 13 (E) $\frac{44}{3}$

21. In rectangle $ABCD$ with area 8, let M be the midpoint of $\overline{AB}$. There exists a point E on $\overline{AD}$ such that $\overline{BE} \perp \overline{CM}$ and $\overline{CE}$ bisects $\angle BED$. What is the area of $\triangle CME$?

- (A) $2\sqrt{2}$ (B) 3 (C) $2\sqrt{3}$ (D) 4 (E) $3\sqrt{2}$

22. A machine has one button labeled $2^a \cdot 3^b$ for each pair of integers $a \geq 0$ and $b \geq 1$. Each minute, the machine shuts off permanently with probability $\frac{1}{21}$. Otherwise, Taiki presses exactly one button, choosing the button labeled $2^a \cdot 3^b$ with probability $2^{-a} \cdot 3^{-b}$. What is the probability that, before the machine shuts off, Taiki has pressed at least three (not necessarily distinct) buttons whose labels are divisible by 36?
- (A) $\frac{125}{2744}$ (B) $\frac{27}{512}$ (C) $\frac{1}{8}$ (D) $\frac{125}{512}$ (E) $\frac{343}{1331}$
23. Ryan writes each of the 12 positive integer divisors of 72 on a whiteboard. Each round, he chooses one of the numbers remaining on the board uniformly at random and erases it along with all of its multiples. The expected number of rounds until the board is empty is $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m + n$?
- (A) 346 (B) 347 (C) 348 (D) 349 (E) 350
24. In $\triangle ABC$ with $AB = 7$, $AC = 8$, and $BC = 9$, let M and N be the midpoints of minor arcs $\widehat{BC}$ and $\widehat{AB}$ of the circumcircle of $\triangle ABC$, respectively. Let B' be the reflection of B across the perpendicular bisector of $\overline{AC}$, and let lines $B'M$ and $B'N$ meet line AC at points P and Q , respectively. Point X satisfies $PX = QX$ and $\angle BAX = \angle CAX$. What is $BX^2 + B'X^2$?
- (A) 44 (B) 46 (C) 48 (D) 50 (E) 52
25. Each of 5 lights is either on or off. Jeff selects 4 of the 10 pairs of lights uniformly at random, and for each selected pair, he is told how many of its two lights are on, but not which ones. What is the expected number of the 32 possible configurations for which the answers Jeff would receive do not uniquely determine which lights are on?
- (A) $\frac{281}{42}$ (B) $\frac{89}{7}$ (C) $\frac{281}{21}$ (D) $\frac{99}{7}$ (E) 16