The JIME Solutions Pamphlet will be available once the contest concludes.

1. **PUBLICATIONS** — For a complete listing of our previous projects, see our website at

   http://mat.mathadvance.org/

2. **Contact Us** — Correspondence about the problems and solutions for this mock AIME and orders for any of our publications should by private message be addressed to dchenmathcounts, innumerateguy, skyscraper, or vvluo.

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*The **Just a Mock Contest** are brought to you by:*

skyscraper, vvluo, LYC, I-_-I, depsilon0, dchenmathcounts, innumerateguy,
and the Mathematical Advancement Committee

MAC JIME

Just a Mock Contest

1st ANNUAL

JIME

Just a Mock Contest

INSTRUCTIONS

1. Do not open this booklet until your proctor gives you the signal to begin.
2. This is a 15-question, 3-hour examination. All answers are integers ranging from 000 to 999, inclusive. Your score will be the number of correct answers; i.e., there is neither partial credit nor a penalty for incorrect answers.
3. No aids other than scratch paper, graph paper, ruler, compass, and protractor are permitted. In particular, calculators and computers are not permitted.
4. A combination of the AIME and the American Mathematics Contest 10/12 are used to determine eligibility for participation in the USA Mathematical Olympiad (USA(J)MO).
5. Record all of your answers, and certain other information, on the AIME answer form. Only the answer form will be collected from you.

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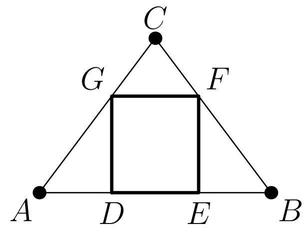
1. Rectangle $DEFG$ is inscribed in the triangle $\triangle ABC$, with sides $AC = 10, AB = 12$, and $BC = 10$. If $AD = CF$, then the length of AD can be expressed as $\frac{m}{n}$ for relatively prime positive positive integers m and n. Find $m + n$.



2. Find the number of ways to partition $\{1, 2, \ldots 12\}$ into three equally-sized sets such that the largest element in each set is divisible by 3. The order of the three sets does not matter.

3. Tom has a collection of 240 books. He takes $\frac{m}{n}$ of the books, for positive integers $m < n$ (not necessarily relatively prime), and then takes $\frac{1}{m}$ of the remaining books. At each instance (before Tom took the books, after he took books for the first time, after he took books for the second time), the number of books remaining formed an arithmetic sequence of integers. Find the sum of all possible values of m.

4. The graph $y = x^2 - \sqrt{2}$ is drawn on a square piece of graph paper bounded by $-100 \leq x \leq 100$ and $-100 \leq y \leq 100$. If the graph paper is divided into 40000 unit squares with equally spaced lines parallel to the sides of the paper intersecting at lattice points, find the number of squares that the graph passes through.

5. Consider a right regular hexagonal prism with all edges congruent. Three vertices of this prism are connected, forming a triangle with an area of $\sqrt{39}$. If the volume of the hexagonal prism is equal to V, find the minimum possible value of V^2.

6. Adam converts a base 10 positive integer n to base 3, and writes it down. Ben sees Adam's result, but interprets it in base 4, and converts it to base 10 to give an integer m. Find the number of possible values of $m - n$ that are at most 2000.

7. For real numbers a, b, and c, suppose that $\frac{a+b}{c}$ and $\frac{b+c}{a}$ are the roots of the polynomial $x^2 - 574x + 17$. Find $\frac{a+c}{b}$.

8. Aaron's math club plans to solve a number of math problems next month. At the end of the month, Aaron will then write the average number of problems each person solved, rounded to the nearest hundredth. He then remarks that the smallest positive number he could not possibly write is 0.08. Find the number of people in his math club.

9. Isosceles triangle ABC has $AB = AC < BC$, incenter I, incircle ω, and inradius 6. Let D be the midpoint of $\overline{BC}$, and let $\overline{AD}$ intersect ω at a point $P \neq D$. The altitude from point B to $\overline{AC}$ intersects $\overline{CI}$ at a point Q. If $PQDC$ is cyclic, find the area of $\triangle ABC$.

10. Right triangle ABC has integer side lengths and right angle at A. The A-excircle is defined as the circle tangent to $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$ on the opposite side of $\overline{BC}$ as A, and its radius is known as the A-exradius. If $\triangle ABC$ has inradius 10, find the sum of all possible values of the A-exradius.

11. Find the number of ordered pairs of positive integers (a, b) such that neither a nor b are divisors of 720, but ab is a divisor of 720^2.

12. Find the sum of all positive integers n such that $\tau(n) \cdot \phi(n) = 3n$, where $\tau(n)$ is the number of divisors of n and $\phi(n)$ is the number of positive integers at most n that are relatively prime to n.

13. In the coordinate plane, an ant begins at the point $(0, 0)$. When the ant is at (x, y), it can move to either $(x + 1, y)$ or $(x, y + 1)$, but cannot move to a point whose coordinates leave the same nonzero remainder when divided by 4. The ant can take a total of N paths to $(68, 68)$. Find the remainder when N is divided by 1000.

14. Find the last three digits of the remainder when $2^{96} - 2^{72} + 2^{28} + 2^{14}$ is divided by $2017 \cdot 2081$.

15. Triangle ABC has side lengths $AB = 9$ and $AC = 15$. Points P and Q are on sides $\overline{AC}$ and $\overline{AB}$ respectively, such that $PB = AB$ and $QC = AC$. If the circumcircle of $\triangle APQ$ passes through the midpoint of $\overline{BC}$, find BC^2.