2021-2022

JMC 10

REMAINS OPEN UNTIL THE DUE DATE

Administration On An Earlier Date Is Not Even Possible

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL. PLEASE READ THE MANUAL BEFORE November 5, 2021.
2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 10 CERTIFICATION FORM (found in the Teachers' Manual) that you followed all rules associated with the conduct of the exam.
3. The Answer Forms must be mailed by trackable mail to the AMC office no later than 24 hours following the exam.
4. *The publication, reproduction or communication of the problems or solutions for this contest during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, email, internet, or media of any type is a violation of the competition rules.*

---

*The **Just a Mock Contest** are brought to you by:*

dchenmathcounts, depsilon0, I-_-I, innumerateguy, LYC, skyscraper, vvluo,
and the Mathematical Advancement Committee

MAC JMC

Just a Mock Contest

3 rd ANNUAL

JMC 10

Just a Mock Contest 10

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL THE TIMER STARTS.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer clearly, edits in submission will not be accepted.
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators, smartwatches, or computing devices are allowed. No problems on the test will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working on the problems. You will have 75 minutes to complete the test.
9. When you finish the exam, sign your name in the space provided on the Answer Form.

The Mathematical Advancement Committee (MAC) reserves the right to re-examine students before deciding whether to grant official status to their scores. Students, regardless of their JMC 10 score, will be invited to the July Invitational Mathematics Examination (JIME).

1. There are $10! = 10 \cdot 9 \cdot 8 \cdots 1$ people in a city. After $9!$ people leave this city, what percent of the city's original population remain?

   (A) 9% **(B)** 10% **(C)** 11% **(D)** 81% **(E)** 90%

2. At a high school, a trio consists of one junior and two seniors. Suppose that 20% of all juniors and 25% of all seniors were assigned to a trio. If no trios overlap, what is the ratio of the number of juniors to seniors?

   (A) $4:15$ **(B)** $2:5$ **(C)** $15:32$ **(D)** $5:8$ **(E)** $5:4$

3. Two integers are erased from the list of positive integers between 1 to 10, inclusive, one of which is 9. The sum of the remaining numbers is a multiple of the sum of the erased numbers. What is the other number erased?

   (A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

4. Three congruent rectangles are glued together by their sides to form a square. The rectangles are then glued together to make a larger rectangle. What is the ratio of the length of the longer side of the larger rectangle to shorter side of the larger rectangle?

   (A) $\frac{1}{9}$ **(B)** $\frac{1}{3}$ **(C)** 1 **(D)** 3 **(E)** 9

5. Suppose a, b, and c are positive real numbers. Which of the following must increase the value of $a^{\left(\frac{b}{c}\right)}$?

   (A) decrease c **(B)** increase b **(C)** decrease a if $b < c$
   (D) increase c if $a < 1$ **(E)** increase b and c

6. Two of the division symbols in the expression

$$1 \div \frac{1}{3} \div \frac{1}{5} \div \frac{1}{2} \div \frac{1}{4}$$

   are changed to multiplication symbols. What is the maximum possible value of the new result?

   (A) $\frac{6}{5}$ **(B)** $\frac{5}{4}$ **(C)** $\frac{15}{8}$ **(D)** $\frac{10}{3}$ **(E)** $\frac{15}{2}$

24. Arjun and Beth are playing a game. Each turn, they choose a card from six distinct face-down cards, each labeled by one of the numbers from 1 to 6, without replacement. At the end, the person with the highest sum of their cards wins. Arjun marked the cards so that he can distinguish between cards, and plays optimally, while Beth randomly selects a card. If Beth goes first, what is the probability that Beth wins?

(A) $\frac{1}{8}$ **(B)** $\frac{7}{48}$ **(C)** $\frac{3}{16}$ **(D)** $\frac{5}{24}$ **(E)** $\frac{1}{4}$

25. What is the area of the region of points (x, y) in the coordinate plane such that $x > 0$ and $3\lfloor x \rfloor + \lceil 4\sqrt{x-y}\, \rceil = 23$? (Recall that $\lfloor r \rfloor$ denotes the greatest integer less than or equal to the real number r, and $\lceil r \rceil$ denotes the smallest integer greater or equal to the real number r.)

(A) 8 **(B)** 9 **(C)** 12 **(D)** 16 **(E)** 21

2021-2022 Mock JMC 10

7. The three longest interior diagonals of a regular hexagon can be used to form the sides of an equilateral triangle with area $4\sqrt{3}$. What is the perimeter of the hexagon?

(A) 9 **(B)** $6\sqrt{3}$ **(C)** 12 **(D)** $12\sqrt{3}$ **(E)** 24

8. All even digits of a four-digit number are replaced by $\diamondsuit$, and all odd digits are similarly replaced by $\heartsuit$. The resulting number is $\heartsuit\diamondsuit\heartsuit\heartsuit$. How many possible values are there for the sum of the digits of this number?

(A) 16 **(B)** 17 **(C)** 18 **(D)** 34 **(E)** 35

9. The vertices of a square are labelled with the numbers 1, 2, 3, and 4. For how many of the conditions below is it possible to label the square accordingly?

- The sum of the numbers of any two adjacent vertices is prime.
- The product of the numbers of any two adjacent vertices is even.
- The sum of the numbers of two non-adjacent vertices is never prime.
- No number is ever twice a number on an adjacent vertex.

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

10. How many integers n satisfy $\lfloor \frac{n^2}{2} \rfloor = \lfloor \frac{n}{2} \rfloor^2$? (Recall that $\lfloor r \rfloor$ denotes the greatest integer less than or equal to the real number r.)

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

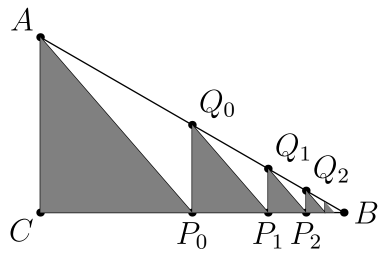
11. The positive integers are listed, starting with 1. The next two integers are listed in backwards order, the next three integers are listed in backwards order, the next four are listed backwards, and so on. If 2021 is the n^{th} term of the sequence, what is n?

(A) 2066 **(B)** 2067 **(C)** 2068 **(D)** 2076 **(E)** 2077

12. Joe has three types of socks, four types of suits, and five types of ties. An outfit is created by one or zero socks, one or zero suits, and one or zero ties. How many possible outfits can Joe create?

(A) 60 **(B)** 72 **(C)** 84 **(D)** 120 **(E)** 144

13. In the diagram below, triangle ABC is right; P_n is located halfway between points P_{n-1} and B where $P_{-1} = C$; Q_n is located directly above P_n. If the area of $\triangle ABC$ is 96, what is the total area of the unshaded regions?



(A) 16 **(B)** 24 **(C)** 32 **(D)** 36 **(E)** 48

14. How many ways are there to choose six distinct integers from $\{1, 2, \ldots, 11\}$ such that it is possible to attach each integer to a face of the cube where the sum of the integers on all pairs of opposite faces are equal?

(A) 24 **(B)** 28 **(C)** 30 **(D)** 50 **(E)** 54

15. Triangle ABC has $AB = 14$ and $AC = 35$. Let E, F, and G be points on sides $\overline{AB}$, $\overline{AC}$, and $\overline{BC}$ respectively such that quadrilateral $AEGF$ is a rhombus. What is the length of segment AE?

(A) 4 **(B)** 6 **(C)** 9 **(D)** 10 **(E)** 12

16. Let n be a three-digit positive integer not divisible by 3 such that it is impossible to form a number divisible by three by removing any single digit from n. How many possible values of n are there?

(A) 148 **(B)** 198 **(C)** 200 **(D)** 216 **(E)** 240

17. George draws a symmetric snowman on a paper. The top snowball is a circle of radius 3, and the middle snowball has radius 4. A line tangent to the top and bottom snowballs is 1 unit away from the circumference of the middle snowball. What is the radius of the bottom snowball?

(A) $\frac{13}{3}$ **(B)** $\frac{41}{9}$ **(C)** $\frac{43}{5}$ **(D)** $\frac{28}{3}$ **(E)** $\frac{48}{5}$

2021-2022 Mock JMC 10

18. Each term x from the sequence of positive integers $1, 2, \ldots, 100$ is replaced with two nonnegative integers a and b to form another sequence, such that $x = a^2 + b$ and a is maximized. How many more digits are in the new sequence than the original sequence?

(A) 25 **(B)** 28 **(C)** 31 **(D)** 34 **(E)** 35

19. The number 2^n has $\frac{n}{3} - 3$ digits and the number 5^n has $\frac{5n}{7} - 1$ digits. What is n?

(A) 42 **(B)** 63 **(C)** 84 **(D)** 105 **(E)** 126

20. Let (a, b, c) be an ordered triple of factors of 1200. How many such triples satisfy $\gcd(a, b, c) \cdot \operatorname{lcm}(a, b, c) = abc$?

(A) 248 **(B)** 280 **(C)** 324 **(D)** 350 **(E)** 364

21. In $\triangle ABC$, the perpendicular bisector of side $\overline{AC}$ intersects side $\overline{BC}$ at point D between B and C. If the circumcenter of $\triangle ADC$ lies on the circumcircle of $\triangle ABC$, $AB = 5$, and $AD = 6$, then what is the area of $\triangle ABC$?

(A) $\frac{67}{3}$ **(B)** 24 **(C)** $\frac{132}{5}$ **(D)** 30 **(E)** $\frac{204}{5}$

22. In the expression $5m + \square$, the box is replaced with an integer so that there is only one integer m that makes $3m - 13$, $5m + \square$, and $2m - 32$ strictly decreasing in that order. How many integers could have been inserted into the box?

(A) 2 **(B)** 4 **(C)** 6 **(D)** 8 **(E)** 10

23. Let $y = ax^2 + bx + c$ be a parabola that passes through $(3, 2021)$ and intersects $y = \frac{2}{3}x$ at two distinct points with integer x-coordinates. What is the maximal possible value of a?

(A) 505 **(B)** 673 **(C)** $\frac{2019}{2}$ **(D)** 1010 **(E)** $\frac{2021}{2}$