2021-2022

JMC 12

REMAINS OPEN UNTIL THE DUE DATE

Administration On An Earlier Date Is Not Even Possible

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL. PLEASE READ THE MANUAL BEFORE November 1, 2021.
2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 12 CERTIFICATION FORM (found in the Teachers' Manual) that you followed all rules associated with the conduct of the exam.
3. The Answer Forms must be mailed by trackable mail to the AMC office no later than 24 hours following the exam.
4. *The publication, reproduction or communication of the problems or solutions for this contest during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, email, internet, or media of any type is a violation of the competition rules.*

*The **Just a Mock Contest** are brought to you by:*

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and the Mathematical Advancement Committee

MAC JMC

Just a Mock Contest

2 nd ANNUAL

JMC 12

Just a Mock Contest 12

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL THE TIMER STARTS.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer clearly, edits in submission will not be accepted.
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators, smartwatches, or computing devices are allowed. No problems on the test will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working on the problems. You will have 75 minutes to complete the test.
9. When you finish the exam, sign your name in the space provided on the Answer Form.

The Mathematical Advancement Committee (MAC) reserves the right to re-examine students before deciding whether to grant official status to their scores. Students, regardless of their JMC 12 score, will be invited to the July Invitational Mathematics Examination (JIME).

1. There are $10! = 10 \cdot 9 \cdot 8 \cdots 1$ people in a city. After $9!$ people leave this city, what percent of the city's original population remain?

   (A) 9% **(B)** 10% **(C)** 11% **(D)** 81% **(E)** 90%

2. Initially, the total mass of red blocks equals the total mass of blue blocks in a pile. A large red block is added to the pile. Now, red blocks consist of 90% of the pile's mass. What is the ratio of the mass of the large red block to the mass of the *initial* pile?

   (A) $2:1$ **(B)** $5:2$ **(C)** $4:1$ **(D)** $9:2$ **(E)** $8:1$

3. Suppose a, b, and c are positive real numbers. Which of the following must increase the value of $a^{\left(\frac{b}{c}\right)}$?

   (A) decrease c **(B)** increase b **(C)** decrease a if $b < c$
   (D) increase c if $a < 1$ **(E)** increase b and c

4. The three longest interior diagonals of a regular hexagon can be used to form the sides of an equilateral triangle with area $4\sqrt{3}$. What is the perimeter of the hexagon?

   (A) 9 **(B)** $6\sqrt{3}$ **(C)** 12 **(D)** $12\sqrt{3}$ **(E)** 24

5. The vertices of a square are labelled with the numbers 1, 2, 3, and 4. For how many of the conditions below is it possible to label the square accordingly?

   - The sum of the numbers of any two adjacent vertices is prime.
   - The product of the numbers of any two adjacent vertices is even.
   - The sum of the numbers of two non-adjacent vertices is never prime.
   - No number is ever twice a number on an adjacent vertex.

   (A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

6. Chen is trying to fit puzzle pieces together, made with the letters A and B, into $ABABABABABAB$. He has the puzzle pieces $ABAB$, ABA, BAB, and AB. In how many ways can he fit together the pieces?

   (A) 3 **(B)** 4 **(C)** 6 **(D)** 7 **(E)** 8

24. What is the area of the region of points (x, y) in the coordinate plane such that $x > 0$ and $3\lfloor x \rfloor + \lceil 4\sqrt{x-y}\, \rceil = 23$? (Recall that $\lfloor r \rfloor$ denotes the greatest integer less than or equal to the real number r, and $\lceil r \rceil$ denotes the smallest integer greater or equal to the real number r.)

(A) 8 **(B)** 9 **(C)** 12 **(D)** 16 **(E)** 21

25. Let ω_1, ω_2 be tangent circles with radii 2 and 4 respectively and point of tangency P. Let D be a point on ω_1 such that $DP = 1$. If the two tangent lines from D to ω_2 intersect ω_1 at X, Y respectively, the length of segment XY can be expressed as $\frac{m\sqrt{n}}{p}$ for positive integers m, n, and p such that m and p are relatively prime and n is square-free. What is $m + n + p$?

(A) 48 **(B)** 52 **(C)** 54 **(D)** 76 **(E)** 92

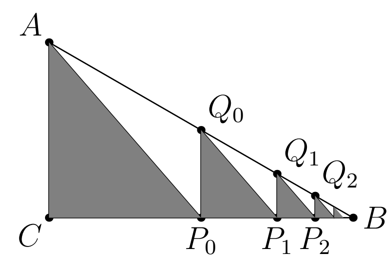
7. In quadrilateral $ABCD$, $\angle DCA = \angle BAC = 90°$, and diagonal $\overline{BD}$ bisects $\angle ADC$. If $AC = 4$ and $CD = 3$, what is the length of $\overline{BD}$?

(A) $2\sqrt{5}$ **(B)** $3\sqrt{3}$ **(C)** $5\sqrt{3}$ **(D)** $4\sqrt{5}$ **(E)** $8\sqrt{2}$

8. Joe has three types of socks, four types of suits, and five types of ties. An outfit is created by one or zero socks, one or zero suits, and one or zero ties. How many possible outfits can Joe create?

(A) 60 **(B)** 72 **(C)** 84 **(D)** 120 **(E)** 144

9. In the diagram below, triangle ABC is right; P_n is located halfway between points P_{n-1} and B where $P_{-1} = C$; Q_n is located directly above P_n. If the area of $\triangle ABC$ is 96, what is the total area of the unshaded regions?



(A) 16 **(B)** 24 **(C)** 32 **(D)** 36 **(E)** 48

10. There exists a unique real number x satisfying $64^{(\log_{16} x)} = 9$. What is the value of x?

(A) $\sqrt[4]{27}$ **(B)** $\sqrt[6]{243}$ **(C)** 3 **(D)** $\sqrt[3]{81}$ **(E)** 81

11. Let n be a three-digit positive integer not divisible by 3 such that it is impossible to form a number divisible by three by removing any single digit from n. How many possible values of n are there?

(A) 148 **(B)** 198 **(C)** 200 **(D)** 216 **(E)** 240

12. Triangle ABC has $AB = 14$ and $AC = 35$. Let E, F, and G be points on sides $\overline{AB}$, $\overline{AC}$, and $\overline{BC}$ respectively such that quadrilateral $AEGF$ is a rhombus. What is the length of segment AE?

(A) 4 **(B)** 6 **(C)** 9 **(D)** 10 **(E)** 12

13. The number 2^n has $\frac{n}{3} - 3$ digits and the number 5^n has $\frac{5n}{7} - 1$ digits. What is n?

(A) 42 **(B)** 63 **(C)** 84 **(D)** 105 **(E)** 126

14. Real numbers x and y are randomly chosen within the interval $[0, \frac{\pi}{2}]$. What is the probability that $\sin^2(x+y) + \cos^2(x) < 1$?

(A) $\frac{1}{4}$ **(B)** $\frac{3}{8}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{5}{8}$ **(E)** $\frac{3}{4}$

15. Each term x from the sequence of positive integers $1, 2, \ldots, 100$ is replaced with two nonnegative integers a and b to form another sequence, such that $x = a^2 + b$ and a is maximized. How many more digits are in the new sequence than the original sequence?

(A) 25 **(B)** 28 **(C)** 31 **(D)** 34 **(E)** 35

16. Let (a, b, c) be an ordered triple of factors of 1200. How many such triples satisfy $\gcd(a, b, c) \cdot \operatorname{lcm}(a, b, c) = abc$?

(A) 248 **(B)** 280 **(C)** 324 **(D)** 350 **(E)** 364

17. Let $ABCD$ be a square with side length 5. Points P, Q, R, and S lie on sides $\overline{AB}, \overline{BC}, \overline{CD}$, and $\overline{AD}$ respectively where $PQRS$ is a square with area 13. What is the maximum possible area of the quadrilateral formed by the pairwise intersections of lines PC, QD, RA, and SB?

(A) $\frac{145}{31}$ **(B)** $\frac{165}{31}$ **(C)** $\frac{165}{29}$ **(D)** $\frac{225}{29}$ **(E)** $\frac{145}{17}$

18. Let z be a complex number satisfying $z^3 = 1$ where $z \neq 1$. What is the real part of

$$\sum_{n=1}^{9} \frac{z + n^2 + 1}{z - n}?$$

(A) $-\frac{81}{2}$ **(B)** $-\frac{55}{2}$ **(C)** $-\frac{45}{4}$ **(D)** $-\frac{25}{4}$ **(E)** $-\frac{5}{2}$

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19. Let $P(x)$ be a monic quadratic with positive integer coefficients. The product of the roots of the quartic $P(P(x)) - 12 \cdot P(x) + 20$ is equal to -4. What is the sum of all possible values of $P(1)$?

(A) 0 **(B)** 2 **(C)** 6 **(D)** 14 **(E)** 16

20. Quadrilateral $ABCD$ has perpendicular diagonals that meet at E. The circumcircles of $\triangle ABE$ and $\triangle CDE$ have centers that are 4 units apart, and intersect again at $F \neq E$ where $\angle ABF = 30°$. What is the area of $ABCD$?

(A) 6 **(B)** $4\sqrt{3}$ **(C)** $8\sqrt{2}$ **(D)** 12 **(E)** $8\sqrt{3}$

21. In the expression $5m + \square$, the box is replaced with an integer so that there is only one integer m that makes $3m - 13$, $5m + \square$, and $2m - 32$ strictly decreasing in that order. How many integers could have been inserted into the box?

(A) 2 **(B)** 4 **(C)** 6 **(D)** 8 **(E)** 10

22. Arjun and Beth are playing a game. Each turn, they choose a card from six distinct face-down cards, each labeled by one of the numbers from 1 to 6, without replacement. At the end, the person with the highest sum of their cards wins. Arjun marked the cards so that he can distinguish between cards, and plays optimally, while Beth randomly selects a card. If Beth goes first, what is the probability that Beth wins?

(A) $\frac{1}{8}$ **(B)** $\frac{7}{48}$ **(C)** $\frac{3}{16}$ **(D)** $\frac{5}{24}$ **(E)** $\frac{1}{4}$

23. A moth resides in a unit equilateral triangle $\triangle ABC$. Denote the moth's position by P, where P is initially the midpoint of AB. As the moth moves, it stays inside the triangle, and PM and PC each never increase, where M is the midpoint of BC. The area of the region of all points that the moth can reach can be expressed in the form $\frac{a\pi - b\sqrt{3}}{c}$ where a, b, and c are positive integers and $\gcd(a, b)$ is relatively prime to c. What is $a + b + c$?

(A) 73 **(B)** 87 **(C)** 195 **(D)** 217 **(E)** 221