KMMC

KARATE MASTERS MATHEMATICS COMPETITIONS

Karate Masters Mathematics Competitions

1st Annual

KMMC 8

Monday, November 9, 2020

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOU DECIDE TO BEGIN.
2. This is a twenty-five question multiple choice test. For each question, only one answer choice is correct.
3. Mark your answer to each problem on the KMMC 8 Answer Form with a keyboard. Check the keys for accuracy and erase errors and stray marks completely.
4. SCORING: You will receive 1 point for each correct answer, 0 points for each problem left unanswered, and 0 points for each incorrect answer.
5. Only blank scratch paper, rulers, and erasers are allowed as aids. Prohibited materials include calculators, smartwatches, phones, computing devices, compasses, protractors, and graph paper. No problems on the competition will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the competition, your competition manager will not ask you to record your name and other information on the answer sheet.
8. You will have 40 minutes to complete the competition once you start the test.
9. When you finish the competition, don't sign your name in the space provided on the answer sheet.

The KMMC Committee reserves the right to disqualify scores from a school if it determines that the rules or the required security procedures were not followed.

The publication, reproduction or communication of the problems or solutions of this competition during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via phone, email, or digital media of any type during this period is a violation of the competition rules.

1. What is the value of
$$20^2 - 0^2 + 0^2 \cdot 1?$$

(A) 0 **(B)** 1 **(C)** 20 **(D)** 400 **(E)** 401

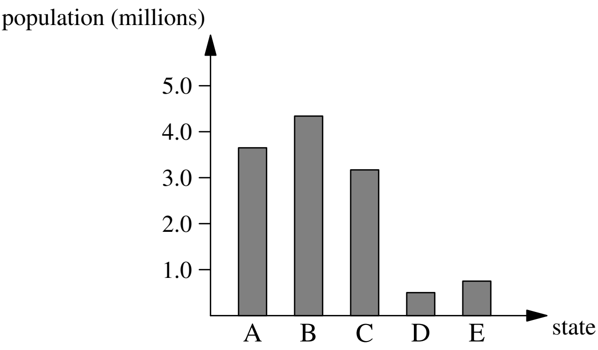
2. How many lines of symmetry does my face have, shown below, if my right eye is six nanometers wider open than my left eye?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

3. Bill says, “All prime numbers are odd." Ben replies, “No. The number n contradicts your statement." What is n?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

4. In the bar graph below, five states are compared in terms of their population. Which of the following is the closest to the difference in population between the most and least populated of the five states, in millions?



(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

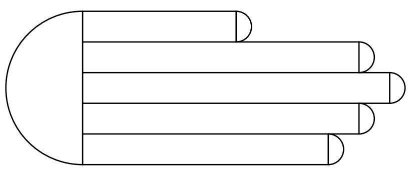
5. What is the value of $1+3+5+\cdots+19+21$?

   (A) 81 **(B)** 91 **(C)** 101 **(D)** 111 **(E)** 121

6. Karate has to feed the terrible Thompson triplets. He has 100 pieces of chicken nuggets in the freezer. The Thompson triplets insist on each having a whole number of pieces such that the ratio of the number of pieces each triplet gets is 3 : 4 : 5. If the Thompson triplets are ravenous and will eat as many pieces as possible, how many pieces will not be eaten by the Thompson triplets?

   (A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

7. A karate chop has been cut into a figure consisting of six semicircles and five rectangles, as shown below. The lengths of the rectangles, from top to bottom, are 5, 9, 10, 9, and 8, and all of the rectangles have a width of 1. What is the area of the karate chop?



   (A) $36+\frac{15\pi}{32}$ **(B)** $36+\frac{15\pi}{16}$ **(C)** $41+\frac{15\pi}{8}$
   (D) $41+\frac{15\pi}{4}$ **(E)** $41+\frac{15\pi}{2}$

8. Farmer Karate wants to find out how many of every animal there are in his farm. However, his eyesight has gotten very bad, so he cannot tell apart his animals, which are simply chicken and sheep. Farmer Karate is clever, so he instead counts the number of feet and then counts the number of heads of his animals. He counts 24 feet and 8 heads. How many chickens does he have?

   (A) 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 8

9. Karate happens to be a teacher of a class of size 21. He gives his class a test, and after the test he realizes that the mean score of the class was a 92 (out of 100). He then gets a new student who takes the test and finds out that the mean is still an integer. If the new student scored at least a 50 but at most a 90, what score did he get?

   (A) 65 **(B)** 70 **(C)** 75 **(D)** 80 **(E)** 85

10. Karate's Rubik's cube broke after his cousin sat on it. Given that it broke evenly into 27 cubes, and the total volume of the Rubik's cube was 27 cubic inches, what is the sum of the surface areas of all 27 cubes, in square inches?

(A) 100 **(B)** 160 **(C)** 162 **(D)** 182 **(E)** 200

11. Two fair six-sided dice are rolled. What is the probability that the square of the sum of the numbers facing up on the dice is divisible by 8?

(A) $\frac{1}{6}$ **(B)** $\frac{2}{9}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{5}{18}$ **(E)** $\frac{1}{3}$

12. Karate lives on a flat plane. He also loves to go on really long runs. He always starts at his house, jogs 2.5 miles north, jogs 6 miles east, and then jogs directly back to his house in a straight line. If Karate runs the first half of his path's distance at 5 miles per hour and then slows down to 2 miles per hour for the last half, how long are his runs, in minutes?

(A) 45 **(B)** 90 **(C)** 180 **(D)** 225 **(E)** 315

13. Now that Halloween is over, Karate has 500 pieces of candy that he still has to eat. He decides to start eating them on Saturday, and he eats 20 pieces of candy each day. On what day of the week will he finish?

(A) Monday **(B)** Tuesday **(C)** Wednesday
(D) Thursday **(E)** Friday

14. If Karate travels at 78 miles per hour for 100 minutes going from his house to the hospital, how many miles per hour would he need to travel in 65 minutes to travel from the hospital back to his house?

(A) 100 **(B)** 110 **(C)** 120 **(D)** 130 **(E)** 140

15. Karate has a bag of marbles, where 3 of them are white, and the rest are black. He draws 4 marbles from the bag at random, all at once. If the probability of drawing 2 white marbles and 2 black marbles is equal to the probability of drawing 1 white marble and 3 black marbles, then how many marbles were in the bag at the start?

(A) 6 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 13

16. How many ordered pairs of positive integers (x, y) satisfy

$$x^4 = y^2 + 8?$$

(A) 0 **(B)** 1 **(C)** 2 **(D)** 4 **(E)** 8

17. How many permutations of the word $KARATE$ are there such that the two A's are not next to each other?

(A) 60 **(B)** 120 **(C)** 180 **(D)** 240 **(E)** 300

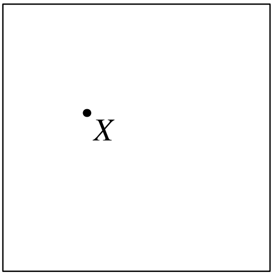
18. Karate has n pieces of candy. Karate realizes that there are an odd number of values of k for which he can split the pieces of candy into k different groups such that each group has an equal number of pieces of candy. If n is between 5 and 500, inclusive, then how many values of n are possible?

(A) 17 **(B)** 18 **(C)** 19 **(D)** 20 **(E)** 21

19. Karate has a whole number of cookies in his bag. If he had 5 more cookies than he currently does, he could give an equal number of cookies to 9 different people with none left over. If he had 2 fewer cookies than he currently does, he could give an equal number of cookies to 8 different people with none left over. Let N be the smallest possible number of cookies in Karate's bag. What is the sum of the digits of N?

(A) 11 **(B)** 12 **(C)** 13 **(D)** 14 **(E)** 15

20. A point X is randomly chosen from the interior of a square with side length 2. What is the probability that X is within 1 unit from the midpoints of at least two sides of the square?



(A) $\frac{1}{2}$ **(B)** $\frac{\pi-1}{4}$ **(C)** $\frac{\pi-2}{2}$ **(D)** $\frac{\pi+2}{8}$ **(E)** $\frac{2\pi-1}{8}$

21. There exists a positive real number x such that

$$x^2 + 4x - 2020 = 0.$$

What is the sum of the digits of the nearest integer to x?

(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

22. Karate wins a bag of candy. He does not know how much candy is inside, but he does know that there are only blue, red, and green candies. He also remembers that the person who gave it to him told him that the probability of pulling a red candy from the bag at random is $\frac{11}{21}$, and the probability of pulling a blue candy from the bag at random is $\frac{13}{45}$. What is the smallest possible number of candies in her bag?

(A) 300 **(B)** 315 **(C)** 330 **(D)** 345 **(E)** 360

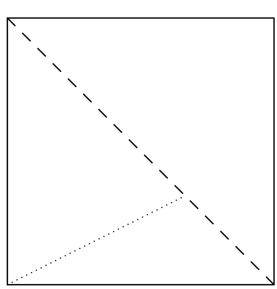
23. If the number $8^a \cdot 9^b$ has 7800 positive integer divisors, where a and b are positive integers, what is the smallest possible value of a?

(A) 3 **(B)** 4 **(C)** 8 **(D)** 13 **(E)** 17

24. Karate writes the first 10 positive perfect squares on a whiteboard. He then uses as many of the digits that he wrote as possible to create a multiple of 9. For example, with the digits 9, 9, 8, 2, and 1, he can create the number 9189. How many digits does Karate use?

(A) 14 **(B)** 15 **(C)** 16 **(D)** 17 **(E)** 18

25. Karate is doing some origami. He starts with a square piece of paper, then folds along the dashed line, and finally, folds along the dotted line. If the extension of the dotted line connects the bottom-left vertex to the midpoint of the right side, what is the ratio of the area of the resulting shape he forms to the area of his original piece of paper?



(A) $\frac{27}{95}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{12}{35}$ **(D)** $\frac{8}{21}$ **(E)** $\frac{29}{70}$

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DO NOT OPEN UNTIL MONDAY, November 9, 2020

****Administration on an earlier date will disqualify your results.****

- All the information needed to administer this exam is not contained in the non-existent KMMC 8 Teacher's Manual. PLEASE READ THE MANUAL BEFORE MONDAY, NOVEMBER 9, 2020.
- Send **Aathreyakadambi**, **DeToasty3**, and **karate7800** a private message submitting your answers to the KMMC 8. AoPS is the only way to submit your answers.
- The publication, reproduction or communication of the problems or solutions of this exam during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via copier, telephone, e-mail, World Wide Web or media of any type during this period is a violation of the competition rules.

For more information about the KMMC and our other competitions, please visit
Wait, we don't have a website!

Questions and comments about this competition should be sent to:

Aathreyakadambi, **DeToasty3**, and **karate7800**.

The problems and solutions for this KMMC 8 were prepared by the KMMC Editorial Board under the direction of:

Aathreyakadambi, DeToasty3, & karate7800