

KMMC

Karate Masters Mathematics Competitions

Karate Masters Mathematics Competitions

3rd Annual

KMMC 10

Sunday, November 1, 2026



INSTRUCTIONS

1. DO NOT TURN TO THE NEXT PAGE UNTIL YOUR COMPETITION MANAGER TELLS YOU TO BEGIN.
2. This is a 25-question multiple-choice competition. For each question, only one answer choice is correct.
3. Mark your answer to each problem on the answer sheet with a #2 pencil. Check blackened answers for accuracy and erase errors completely. Only answers that are properly marked on the answer sheet will be scored.
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. Only blank scratch paper, rulers, compasses, and erasers are allowed as aids. No calculators, smartwatches, phones, or computing devices are allowed. No problems on the competition will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the competition, your competition manager will ask you to record your name on the answer sheet.
8. You will have 75 minutes to complete the competition once your competition manager tells you to begin.
9. When you finish the competition, sign your name in the space provided on the answer sheet and complete the demographic information questions on the back of the answer sheet.

The problems and solutions for this KMMC 10 were prepared
by the KMMC Committee under the direction of:

Taiki Aiba and Ryan Shin

The KMMC Committee reserves the right to disqualify scores from a school if it determines that the rules or the required security procedures were not followed.

The publication, reproduction or communication of the problems or solutions of this competition during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via phone, email, or digital media of any type during this period is a violation of the competition rules.

Students who score well on this KMMC 10 will be invited to take the Karate Masters Invitational Mathematics Examination (KMIME) on Wednesday, December 16, 2026. More details about the KMIME can be found on our site.

1. What is the value of
$$2^6\left(2^2\cdot 6+2+6+2^{2-6}\right)-26?$$

(A) 2018 **(B)** 2026 **(C)** 2038 **(D)** 2096 **(E)** 2162

2. The average (arithmetic mean) of five numbers is 12. When the greatest number is removed, the average of the numbers is 10. When instead the least number is removed, the average of the numbers is 13. What is the sum of the greatest and least of the five numbers?

(A) 22 **(B)** 24 **(C)** 26 **(D)** 28 **(E)** 30

3. Karate writes some kanji characters, each with between 5 to 7 strokes, inclusive. The total number of strokes across all characters is 23. How many characters did Karate write?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

4. Below is the 8×8 multiplication table. How many of its 64 numbers are prime?

1	2	3	4	5	6	7	8
2	4	6	8	10	12	14	16
3	6	9	12	15	18	21	24
4	8	12	16	20	24	28	32
5	10	15	20	25	30	35	40
6	12	18	24	30	36	42	48
7	14	21	28	35	42	49	56
8	16	24	32	40	48	56	64

(A) 8 **(B)** 10 **(C)** 12 **(D)** 14 **(E)** 16

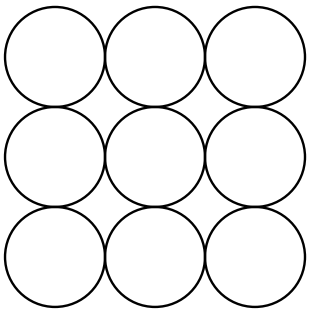
5. Let K, A, R, A', T, and E be positive integers such that $E=A$, $K=A'$, and $R=T$. What is the greatest possible value of $K\cdot A\cdot R\cdot A'\cdot T\cdot E$ less than 1000?

(A) 729 **(B)** 768 **(C)** 900 **(D)** 945 **(E)** 961

6. How many positive integer divisors of $2^{20}\cdot 5^{26}$ have a nonzero units digit?

(A) 47 **(B)** 93 **(C)** 130 **(D)** 283 **(E)** 520

7. Nine congruent circles are arranged in a 3×3 grid so that each circle is tangent to the circles adjacent to it. How many lines are tangent to at least three of the nine circles?



(A) 8 **(B)** 10 **(C)** 12 **(D)** 14 **(E)** 16

8. What is the value of

$$1^2 - 2^2 - 3^2 + 4^2 + 5^2 - 6^2 - 7^2 + 8^2 + \cdots + 2025^2 - 2026^2 - 2027^2 + 2028^2?$$

(A) -2028 **(B)** 1014 **(C)** 2027 **(D)** 2028 **(E)** 4055

9. Karate drives to martial arts class in three legs. His speeds on the legs form an increasing arithmetic sequence, and the lengths of the legs form an increasing geometric sequence. The first leg is 20 miles at 20 miles per hour, the second leg is 40 miles, and the third leg is driven at 60 miles per hour. What is Karate's average speed, in miles per hour, for the whole trip?

(A) 35 **(B)** 36 **(C)** 40 **(D)** 42 **(E)** 45

10. Two vertical trees stand on level ground. The first has foot M, top A, and height 65. The second has foot T, top H, and height 33. Also, $\angle MHA = 90^\circ$. What is MT?

(A) $\sqrt{66}$ **(B)** 16 **(C)** $2\sqrt{66}$ **(D)** 32 **(E)** $4\sqrt{66}$

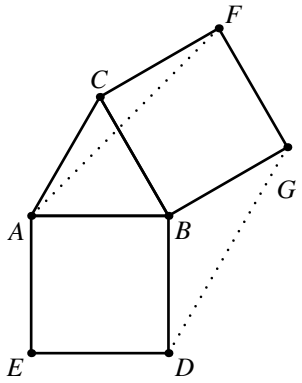
11. How many ordered pairs (a, b) of positive integers with $a < b$ satisfy

$$\operatorname{lcm}(a, b) - \gcd(a, b) = 2026,$$

where $\gcd(a, b)$ denotes the greatest common factor of a and b, while $\operatorname{lcm}(a, b)$ denotes their least common multiple?

(A) 4 **(B)** 6 **(C)** 7 **(D)** 9 **(E)** 11

12. The following figure is formed from two squares erected outside an equilateral triangle. What is the degree measure of the acute angle formed by lines AF and DG?

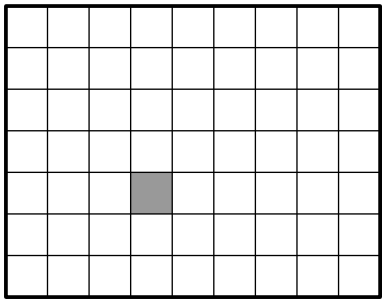


(A) 10 **(B)** 15 **(C)** 20 **(D)** 25 **(E)** 30

13. How many ordered quadruples (M, A, T, H) of positive integers with $M > 1$ and $T > 1$ satisfy $M^A \cdot T^H = 48$?

(A) 16 **(B)** 18 **(C)** 20 **(D)** 24 **(E)** 32

14. A 7×9 rectangle is divided into 63 unit squares by grid lines, and one unit square is shaded, as shown. Two of the interior 8 vertical grid lines and two of the interior 6 horizontal grid lines are chosen, and the four lines divide the rectangle into nine smaller rectangles. In how many ways can the four lines be chosen so that the shaded square lies in the center rectangle?



(A) 60 **(B)** 96 **(C)** 120 **(D)** 225 **(E)** 360

15. Chord $\overline{AB}$ of length 8 is drawn in a circle with center O and radius 5. The set of all points P on the circle for which there is another point Q on the circle such that $\overline{AB}$ bisects chord $\overline{PQ}$ forms an arc of the circle with endpoints X and Y. What is the area of $\triangle OXY$?

(A) $2\sqrt{6}$ **(B)** $5\sqrt{3}$ **(C)** $2\sqrt{21}$ **(D)** $4\sqrt{6}$ **(E)** 12

16. A subset S of $\{1, 2, 3, \ldots, 30\}$ has the property that no element of S is exactly 3 times another element of S. How many such subsets have the greatest possible number of elements?

    (A) 32 **(B)** 64 **(C)** 96 **(D)** 128 **(E)** 192

17. Ayaka and Judo take turns rolling a fair six-sided die. Ayaka goes first, and the first player to roll a number already rolled by either player loses. What is the probability that Ayaka wins?

    (A) $\frac{5}{11}$ **(B)** $\frac{155}{324}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{169}{324}$ **(E)** $\frac{6}{11}$

18. Let f be a function defined on the set of nonnegative integers such that $f(1) = 1$, and

$$\begin{cases} f(3n) = 2f(n) \\ f(3n+1) = f(3n+2) = 2f(n) + 1 \end{cases}$$

    for all nonnegative integers n. How many positive integers m satisfy $f(m) = 2026$?

    (A) 128 **(B)** 160 **(C)** 192 **(D)** 240 **(E)** 256

19. Circle ω has radius 7. Chord $\overline{PQ}$ of ω contains a point R with $PR = 3$ and $QR = 6$. Two smaller circles lie on opposite sides of line PQ. Each is tangent to $\overline{PQ}$ at R and internally tangent to ω. What is the product of the radii of the two smaller circles?

    (A) 2 **(B)** $\frac{18}{7}$ **(C)** $\frac{7}{2}$ **(D)** 4 **(E)** $\frac{9}{2}$

20. On a circular track of length 400 meters, Karate runs counterclockwise at 4.1 meters per second, and Judo and Aki run clockwise at 2.2 and 6.4 meters per second, respectively. They begin running at the same time from the same point on the track. At how many times during the first hour, including the start, do at least two of them meet at the same point on the track?

    (A) 133 **(B)** 151 **(C)** 152 **(D)** 171 **(E)** 190

21. Isosceles trapezoid $ABCD$, whose diagonals meet at O, has $AB = 126$, $CD = 42$, $\overline{AB} \parallel \overline{CD}$, and height 92. Let A_2 be the foot of the perpendicular from C to line AD, and let B_2 be the foot of the perpendicular from D to line BC. For $k \geq 2$, let A_{k+1} be the foot of the perpendicular from B_k to line AD, and let B_{k+1} be the foot of the perpendicular from A_k to line BC. Both sequences of points approach a point P. What is OP?

    (A) 23 **(B)** 46 **(C)** 69 **(D)** 92 **(E)** 138

22. Let $\lfloor x \rfloor$ denote the greatest integer less than or equal to x, and let $\lceil x \rceil$ denote the least integer greater than or equal to x. For how many positive integers $n \leq 2026$ does

$$\lfloor 2x \rfloor + \lceil 2x \rceil + \lfloor 3x \rfloor + \lceil 3x \rceil = n$$

have infinitely many real solutions x?

(A) 609 **(B)** 810 **(C)** 811 **(D)** 1013 **(E)** 1621

23. In a carnival game, Karate draws cards one at a time with replacement. Each card shows an integer from 1 to 26, inclusive, with each integer equally likely to be drawn. He keeps a running total of the numbers drawn and stops as soon as the total is at least 26. He wins if the total is exactly 26 when he stops. The probability that Karate wins is $\frac{m}{n}$, where m and n are relatively prime positive integers. How many positive integer divisors does m have?

(A) 26 **(B)** 27 **(C)** 73 **(D)** 76 **(E)** 79

24. For nonnegative integers a and b, let $a \oplus b$ be the result of adding a and b in base 2 without carrying: each digit of $a \oplus b$ is 1 if the corresponding digits of a and b differ, and 0 otherwise. For example, $5 \oplus 3 = 101_2 \oplus 011_2 = 110_2 = 6$. How many integers can be written as

$$\frac{a+b+c}{a \oplus b \oplus c}$$

for positive integers a, b, and c less than or equal to 63?

(A) 64 **(B)** 80 **(C)** 95 **(D)** 96 **(E)** 127

25. A country has 12 cities, two of which are Haru and Fuyu. Each of the other 10 cities has one road to Haru and one road to Fuyu, and there are no other roads. A route between two cities is a sequence of roads, each starting where the previous one ends, that leads from one of the cities to the other. How many sets of roads have the property that removing all of the roads in the set leaves some two cities with no route between them, but adding back any one of those roads gives every two cities a route between them again?

(A) 522 **(B)** 1024 **(C)** 1032 **(D)** 1034 **(E)** 2058