Toasty's Problems

DeToasty3

January 5, 2026

Credits

This document uses the package `shen.sty` by the AoPS user **TheUltimate123**.

Introduction

This handout contains every problem I have proposed that has appeared in a released math competition, mock or official. Note that this also means that I will not be putting problems here that exist outside of a contest or are currently not public.

The math competitions I have written problems for so far are:

- Lexington Math Tournament (LMT)
- Online Test Seasonal Series (OTSS)
- De Mathematics Competitions (DMC)
- Karate Masters Mathematics Competitions (KMMC)
- Karate Masters Mathematics Competitions 2 (KMMC 2)
- ALP x DMC Mathematics Competition (ADMC)
- Geometry AMC (GAMC)
- Mock MATHCOUNTS State (hosted by the AoPS user **smartatmath**)
- berfoer
- Pi (hosted by the AoPS user **PhunsukhWangdu**)
- ANOTHER MOCK CONTEST :O
- sgossk (hosted by the AoPS user **pog**)
- Winter MATHCOUNTS Competition (WMC) (hosted by the AoPS user **peace09**)
- mathleague.org (**Note:** These problems will not be listed in this handout for confidentiality.)
- American Mathematics Competitions (AMC)
- MATHCOUNTS (**Note:** These problems will not be listed in this handout for confidentiality.)

These problems will be listed in chronological order and in the order in which the problems are listed in their respective tests. Additionally, along the way I will be providing some remarks to certain problems, such as how I thought of the problem idea, how much I personally like the problem, and possibly other points that I would like to address. This document will not contain any solutions to my problems, but if you are curious, you can easily find most of the solutions to my problems by using the clickable links above. Finally, I will be updating this document as I continue to propose problems for math competitions.

Without further ado, please sit back and enjoy the problems! (and yes, this was a reference to the AoPS user **Binomial-theorem**)

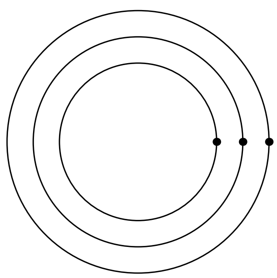
Problems (2019)

1. **(2019 Fall LMT Individual Round P6)**
   Find the minimum possible value of the expression $|x+1|+|x-4|+|x-6|$.

2. **(2019 Fall LMT Individual Round P11)**
   A two-digit number has the property that the difference between the number and the sum of its digits is divisible by the units digit. If the tens digit is 5, how many different possible values of the units digit are there?

3. **(2019 Fall LMT Theme Round P3)**
   Joe Quigley has 12 students in his math class. He will distribute N worksheets among the students. Find the smallest positive integer N for which any such distribution of the N worksheets among the 12 students results in at least one student having at least 3 worksheets.

4. **(2019 Fall LMT Theme Round P7)**
   Three planets with coplanar, circular, and concentric orbits are shown on the backside of this page[1]. The radii of the three circles are 3, 4, and 5. Initially, the three planets are collinear. Every hour, the outermost planet moves one-sixth of its full orbit, the middle planet moves one-fourth of its full orbit, and the innermost planet moves one-third of its full orbit (A full orbit occurs when a planet returns to its initial position). Moreover, all three planets orbit in the same direction. After three hours, what is the area of the triangle formed by the planets as its three vertices?



5. **(2019 Fall LMT Theme Round P11)**
   Festivus occurs every year on December 23rd. In 2019, Festivus will occur on a Monday. On what day will Festivus occur in the year 2029?

> **Remark.** This is the edited version of my problem; I do not have the original problem at hand.

6. **(2019 Fall LMT Theme Round P13)**
   How many permutations of the word $CHRISTMAS$ are there such that the S's are not next to each other and there is not a vowel anywhere between the two S's?

[1] In the official test sheet, the diagram was indeed on the backside of the page.

7. **(2019 Fall LMT Team Round P2)**
Determine the number of positive integers n with $1 \leq n \leq 400$ that satisfy the following:

- n is a square number.
- n is one more than a multiple of 5.
- n is even.

8. **(2019 Fall LMT Guts Round P9)**
A positive integer n is equal to one-third the sum of the first n positive integers. Find n.

9. **(2019 Fall LMT Guts Round P12)**
Define a sequence recursively by $F_0 = 0$, $F_1 = 1$, and for all $n \geq 2$, $F_n = \left\lceil \frac{F_{n-1}+F_{n-2}}{2} \right\rceil + 1$, where $\lceil r \rceil$ denotes the least integer greater than or equal to r. Find F_{2019}.

> **Remark.** The original problem asked for F_{13}, which is something that the AoPS user **GammaZero** will likely understand.

Problems (2020)

1. **(Season 1 TMC 10A P23/12A P20)**
In convex quadrilateral $ABCD$, $\angle A = 90^\circ$, $\angle C = 60^\circ$, $\angle ABD = 25^\circ$, and $\angle BDC = 5^\circ$. Given that $AB = 4\sqrt{3}$, find the area of quadrilateral $ABCD$.

$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 4\sqrt{3} \qquad \textbf{(C)}\ 8 \qquad \textbf{(D)}\ 8\sqrt{3} \qquad \textbf{(E)}\ 16\sqrt{3}$

> **Remark.** This problem was originally proposed to the 2020 Spring LMT. However, due to certain changes, only the math team captains were able to propose problems for that contest, so I ended up moving this problem to OTSS.

2. **(Season 1 TMC 12B P7)**
Given that x and y are positive real numbers, with x and y each less than $\frac{\pi}{2}$, that satisfy the equations $x + y = \frac{\pi}{2}$ and $\sin(x) + 2\cos(y) = \frac{3\sqrt{3}}{2}$, what is $|x - y|$?

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ \frac{\pi}{12} \qquad \textbf{(C)}\ \frac{\pi}{6} \qquad \textbf{(D)}\ \frac{\pi}{4} \qquad \textbf{(E)}\ \frac{\pi}{3}$

3. **(Season 1 TMC 10B P13)**
For Color Day, 12 students in a class are to be randomly assigned a T-shirt to wear with one of three colors: red, blue, and yellow. A color may be worn by as few as 0 students. However, since the teacher wants color balance, there cannot be more than 9 students wearing the same color. In how many ways can this happen? Assume that the students are indistinguishable.

$\textbf{(A)}\ 71 \qquad \textbf{(B)}\ 73 \qquad \textbf{(C)}\ 79 \qquad \textbf{(D)}\ 85 \qquad \textbf{(E)}\ 91$

> **Remark.** This problem was originally proposed to the 2019 Fall LMT for the theme round, where instead of colors, it was which car of the train students would get put in to go to PUMaC. However, this problem was not used, so I ended up moving this problem to OTSS.

4. **(Season 1 TMC 10B P15)**
Let $\triangle ABC$ be isosceles with $AB = AC$. Let D be the reflection of B across the centroid of the triangle and M be the midpoint of $\overline{BC}$. If the area of quadrilateral $ADMB$ is 6 and $BC = 4$, then what is the square of length AB?

$\textbf{(A)}\ 10 \qquad \textbf{(B)}\ 11 \qquad \textbf{(C)}\ 12 \qquad \textbf{(D)}\ 13 \qquad \textbf{(E)}\ 14$

> **Remark.** A previous problem was written by the AoPS user **PCChess**, who used something with the centroid in his problem. However, I took it for a spin and made my own completely different problem that uses area ratios.

5. **(Season 1 TMC 12B P20)**
Richard writes the quadratic $f(x) = ax^2+bx+c$ on a whiteboard, where a, b, and c are distinct nonzero complex numbers. Matthew sees Richard's quadratic, and rearranges the order of the coefficients (i.e. permutes the order of a, b, and c) to make his own six distinct quadratics: $g_1(x)$, $g_2(x)$, $g_3(x)$, $g_4(x)$, $g_5(x)$, and $g_6(x)$ (one of which is equal to $f(x)$). What is the minimum number of possible distinct roots of
$$\prod_{k=1}^{6}(f(x)+g_k(x))?$$

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 10

> **Remark.** I originally wrote a version of this problem that was very guessable and not very full-fledged. However, thanks to the AoPS user **P_Groudon**, he was able to turn my problem into this pretty complete and nice problem, albeit a bit too hard for a P20 on an AMC 12.

6. **(Season 1 TMC 10B P23/12B P21)**
In a room with 10 people, each person knows exactly 4 different languages. A conversation is held between every pair of people with a language in common. If a total of 36 different languages are known throughout the room, and no two people have more than one language in common, what is the sum of all possible values of n such that a total of n conversations are held?

(A) 19 **(B)** 25 **(C)** 32 **(D)** 35 **(E)** 37

> **Remark.** A previous problem was written by the AoPS user **Qinghan04**, who used something with languages and conversations in her problem (her problem is not public, as it was an unused proposal to the 2019 Fall LMT). I expanded upon her promising problem idea and created what I personally believe to be one of the best problems that I have written thus far.

7. **(Season 1 TMC 10B P25/12B P24)**
Let O_1 be a circle with radius r. Let O_2 be a circle with radius between $\frac{r}{2}$ and r, exclusive, that goes through the center of circle O_1. Denote points X and Y as the intersections of the two circles. Let P be a point on the major arc $\widehat{XY}$ of O_1. Let $\overline{PX}$ intersect O_2 at A, strictly between P and X. Let $\overline{PY}$ intersect O_2 at B, strictly between P and Y. Let E be the midpoint of $\overline{PX}$ and F be the midpoint of $\overline{PY}$. If $AY = 100$, $AB = 65$, and $EF = 52$, what is BX?

(A) 104 **(B)** 105 **(C)** 106 **(D)** 107 **(E)** 108

> **Remark.** This problem was originally written by the AoPS user **jeteagle**, who intended this problem to be a problem with coordinate bash as its intended solution. However, me and the AoPS user **P_Groudon** worked our way to include some more interesting geometric concepts such as cyclic quadrilaterals, which mitigated the concern of this problem being too coordinate bash-able.

8. **(Season 1 OTIE P2)**
Jela and Benn are playing a game. Each round, Jela and Benn each flip a fair coin at the same time. Jela and Benn win if they flip heads together. However, they lose if they flip tails together for three rounds in a row. If neither event happens after the end of 4 rounds, they also lose. The probability that Jela and Benn win can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

> **Remark.** Originally, this problem was written by the AoPS user **ivyzheng**, but I came in and the two of us continued to improve the problem to its current state. I do believe, however, that it is a bit too casework-heavy and slightly bashy for a P2 on an AIME. Also, the AoPS user **l-_-l** suggested the names.

9. **(Season 1 OTIE P3)**
Two dogs, Otie and Amy, are each given an integer number of biscuits to eat, where Otie and Amy get x and y biscuits, respectively, and $0 < x < y < 72$. At the start, the numbers x, y, and 72 form an arithmetic progression, in that order. Each dog then eats N of their biscuits, where N is a positive integer less than x. After they finish eating, Amy now has exactly three times the number of biscuits left over as Otie. Find the number of possible values of N.

10. **(Mock MATHCOUNTS State Sprint P3)**
At his current speed, Michael runs 10 yards in 5 seconds. If he runs 0.3 yards per second faster, then how many yards can he run in 10 seconds?

11. **(2021 Spring DMC 10 P1)**
What is the value of
$$\frac{(2^0 - 2^1)^{2020}}{(2 \cdot 0 + 2^0)^{2021}}?$$

$\textbf{(A)}\ -1 \qquad \textbf{(B)}\ -\frac{1}{2} \qquad \textbf{(C)}\ 0 \qquad \textbf{(D)}\ \frac{1}{2} \qquad \textbf{(E)}\ 1$

12. **(2021 Spring DMC 10 P2)**
If the ratio of males to females in a country club is exactly 9 to 5, and there are fewer than 100 people in the club, what is the largest possible number of people in the club? (Assume that all of the people in the club are either male or female.)

$\textbf{(A)}\ 95 \qquad \textbf{(B)}\ 96 \qquad \textbf{(C)}\ 97 \qquad \textbf{(D)}\ 98 \qquad \textbf{(E)}\ 99$

> **Remark.** This problem was originally proposed to the 2019 Fall LMT.

13. **(2021 Spring DMC 10 P4)**
A dog has four legs, and a dug has three legs. Janelle has a whole number of dogs and dugs as pets, and she has no other pets. If there are 61 legs across all of Janelle's pets, what is the smallest possible number of dugs that Janelle could have?

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ 3 \qquad \textbf{(E)}\ 4$

14. **(2021 Spring DMC 10 P5)**
Rohan wants to distribute 25 slices of pizza to n people such that each person gets an equal number of slices, except for one person who gets one more slice than each of the other people. If n is greater than 1, how many different integer values of n exist?

(A) 2 **(B)** 5 **(C)** 7 **(D)** 8 **(E)** 9

> **Remark.** This problem was originally proposed to the Season 2 TMC 10.

15. **(2021 Spring DMC 10 P6)**
Two distinct elements x and y are chosen from the set $\{1, 2, 3, 4\}$ at random. What is the probability that the line with slope $\frac{y}{x}$ passing through the point (x, y) also passes through the point $(2020, 1010)$?

(A) $\frac{1}{12}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{2}$

> **Remark.** This problem was originally proposed to the Season 2 TMC 10.

16. **(2021 Spring DMC 10 P7)**
Anthony, Daniel, and Richard have 17, 20, and 26 trading cards, respectively. Every minute, one of the three boys gives away two of his trading cards such that the other two boys get one trading card each. What is the shortest amount of time, in minutes, that it could take for the three boys to each have an equal number of trading cards?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

> **Remark.** For some reason, I am actually quite proud of writing this problem. I believe that it is actually quite original for a problem in the first ten on an AMC 10.

17. **(2021 Spring DMC 10 P8)**
Two distinct points A and B are chosen on the circumference of a circle with center O. Another point C, distinct from A and B, is chosen on the circumference. If $\angle AOB = 70^\circ$, what is the probability that $\triangle ABC$ is acute?

(A) $\frac{7}{36}$ **(B)** $\frac{7}{18}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{11}{18}$ **(E)** $\frac{31}{36}$

> **Remark.** This problem was originally proposed to the Season 2 TMC 10.

18. **(2021 Spring DMC 10 P9)**
Alice and Bob are racing each other on a track. Each of their lanes are 400 meters in length. Normally, Alice and Bob run at constant rates of a and b meters per minute, respectively, but Alice's lane has a 180-meter sand region in the middle, in which she runs at three-quarters of her normal speed. If Alice and Bob take the same amount of time to run through their lanes without stopping, what is $\frac{a}{b}$?

(A) 1.05 **(B)** 1.15 **(C)** 1.25 **(D)** 1.35 **(E)** 1.45

19. **(2021 Spring DMC 10 P10)**
What is the largest integer n for which there exists an ordered triple (p, q, r) of distinct prime numbers such that $p^2(q^2 + r^2)$ is divisible by 2^n?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

20. **(2021 Spring DMC 10 P12)**
Let A and B be two distinct points on a plane. Let $\mathcal{S}$ denote the set of all circles on the plane with a finite area such that A and B are on the circumference of the circle. What is the region of all points not on the circumference of any of the circles in $\mathcal{S}$?

(A) Every point on line AB excluding A and B
(B) Every point on segment $\overline{AB}$ excluding A and B
(C) Every point on line AB but not on segment $\overline{AB}$
(D) The midpoint of segment $\overline{AB}$
(E) None of the above

> **Remark.** This problem was originally proposed to the Season 2 TMC 10.

21. **(2021 Spring DMC 10 P13)**
10 students are taking a final exam. Of the 10 students, 3 of them are guaranteed to pass. However, the other 7 students are lazy and are not guaranteed to pass, but each of them has the same probability of passing as one another, where the probability is nonzero. If Tomo is one of the 7 lazy students, and exactly 6 out of the 10 students passed the exam, what is the probability that Tomo was one of those 6 students?

(A) $\frac{5}{16}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{2}{5}$ **(D)** $\frac{3}{7}$ **(E)** $\frac{5}{9}$

> **Remark.** This problem was originally proposed to the 2019 Fall LMT. In fact, this was the first problem I have ever written for LMT and the first problem I have ever written that was on a released math contest.

22. **(2021 Spring DMC 10 P17)**
8 people randomly split into 2 groups of four to dance. After that, the 8 people randomly split into 4 pairs of two to talk. What is the probability that exactly 2 of the 4 pairs contain two people who have danced in the same group of four?

(A) $\frac{8}{35}$ **(B)** $\frac{2}{5}$ **(C)** $\frac{4}{21}$ **(D)** $\frac{24}{35}$ **(E)** $\frac{6}{7}$

> **Remark.** A previous problem was written by the AoPS user **ivyzheng**, who used something with grouping and probability in her problem (her problem is not public, as it was an unused proposal to Season 2 OTSS). I simplified her idea to this problem, and I think it works fine as a P17 on an AMC 10.

23. **(2021 Spring DMC 10 P18)**
A plane cuts into a sphere of radius 11 such that the area of the region of the plane inside the sphere is 108π. A perpendicular plane cuts into the sphere such that the area of the region of the plane inside the sphere is 94π. Given that the two planes intersect at a line, what is the length of the segment of the line inside the sphere?

(A) $6\sqrt{3}$ **(B)** 12 **(C)** $11\sqrt{2}$ **(D)** $8\sqrt{5}$ **(E)** 18

> **Remark.** For some reason, I am actually quite proud of writing this problem. I believe that it is actually quite original despite it seeming like something that could have easily appeared before. This is probably my second favorite problem on the test, only behind P23.

24. **(2021 Spring DMC 10 P19)**
Let the sum of $n \geq 2$ consecutive integers be a positive prime number, where the smallest of the integers is a. If $a + n = 28$, what is the sum of all possible values of a?

(A) -26 **(B)** -25 **(C)** -1 **(D)** 0 **(E)** 1

> **Remark.** This problem was fun to make, and this was seen as many users' favorite problem on the test, alongside P23.

25. **(2021 Spring DMC 10 P20)**
In trapezoid $ABCD$ with $\overline{AD} \parallel \overline{BC}$ and side lengths $AD = 18$, $BC = 20$, and $AB = CD = 8$, let X be the intersection of line AB and the bisector of $\angle ADC$, and let Y be the intersection of line CD and the bisector of $\angle DAB$. What is XY?

(A) 22 **(B)** 24 **(C)** 25 **(D)** 27 **(E)** 28

> **Remark.** I tried this problem after several months of not looking at it during a mini-event at the 2021 Spring LMT. I completely embarrassed myself by literally not knowing how to solve my own problem and looking like a fool in front of the AoPS users **pog** and **richy**.

26. **(2021 Spring DMC 10 P21)**
A set of positive integers exists such that for any integer k in the set, all of the values $k^2 + 2$, $k^2 + 4$, and $k^2 + 8$ are prime numbers. Two distinct integers m and n are chosen from the set. Which of the following is a possible value of $m + n$?

(A) 40 **(B)** 56 **(C)** 72 **(D)** 88 **(E)** 104

> **Remark.** Let's just say that creating this problem was much harder than actually solving it. I actually had to manually verify that there exist m and n that satisfy the problem's conditions. Furthermore, I believe that this problem was misplaced on the test and should have been no later than P18 or so.

27. **(2021 Spring DMC 10 P23)**
Joy picks an integer n from the interval $[1, 40]$. She tells Amy the remainder when n is divided by 7 and Sid the number of divisors of n. Amy and Sid both know n is in the interval $[1, 40]$, but they get confused and believe Amy was told the number of divisors and Sid was told the remainder. Amy says, "I know what n is." Sid replies, "If so, then I also know what n is." As it turns out, they thought of the same value but were wrong due to their confusion. If Amy and Sid tell the truth based on their beliefs and can reason perfectly, what is the sum of all possible actual values of n?

(A) 21 **(B)** 22 **(C)** 23 **(D)** 24 **(E)** 25

> **Remark.** This was a problem that I originally thought was long, contrived, and annoying, but later got quite positive feedback, with many users calling this and P19 their two favorite problems on the test.

28. **(2021 Spring DMC 10 P24)**
In triangle ABC, $AB = 16$ and $BC = 8$, with a right angle at C. Let M be the midpoint of side $\overline{AB}$, let N be a point on side $\overline{AC}$, and let P be the intersection of segments $\overline{BN}$ and $\overline{CM}$. If $BP = 7$, what is the sum of all possible values of $\frac{CN}{AN}$?

(A) $\frac{23}{21}$ **(B)** $\frac{21}{19}$ **(C)** $\frac{19}{17}$ **(D)** $\frac{17}{15}$ **(E)** $\frac{15}{13}$

29. **(2020 KMMC 8 P1)**
What is the value of

$$20^2 - 0^2 + 0^2 \cdot 1?$$

(A) 0 **(B)** 1 **(C)** 20 **(D)** 400 **(E)** 401

30. **(2020 KMMC 8 P2)**
How many lines of symmetry does my face have, shown below, if my right eye is six nanometers wider open than my left eye?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

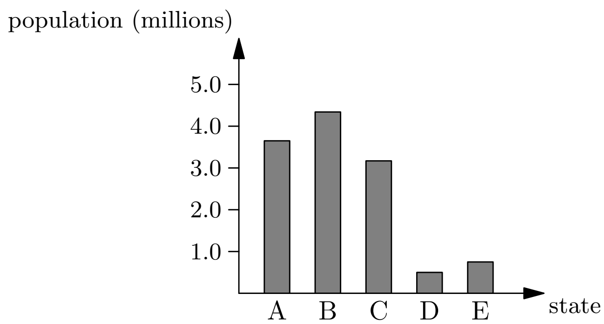
> **Remark.** OMG THIS IS THE MOST BEAUTIFUL PROBLEM I HAVE EVER LAID MY PUPILS UPON IN THE HISTORY OF MATHEMATICAL PROBLEMS!!!! /s

31. **(2020 KMMC 8 P3)**
Bill says, "All prime numbers are odd." Ben replies, "No. The number n contradicts your statement." What is n?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

> **Remark.** This problem was originally proposed to the 2019 Fall LMT as a joke.

32. **(2020 KMMC 8 P4)**
In the bar graph below, five states are compared in terms of their population. Which of the following is the closest to the difference in population between the most and least populated of the five states, in millions?



(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

> **Remark.** This problem was originally proposed to Season 2 OTSS for the never-released TMC 8.

33. **(2020 KMMC 8 P5)**
What is the value of $1 + 3 + 5 + \cdots + 19 + 21$?

(A) 81 **(B)** 91 **(C)** 101 **(D)** 111 **(E)** 121

> **Remark.** This problem was originally proposed to Season 2 OTSS for the never-released TMC 8.

34. **(2020 KMMC 8 P6)**
Karate has to feed the terrible Thompson triplets. He has 100 pieces of chicken nuggets in the freezer. The Thompson triplets insist on each having a whole number of pieces such that the ratio of the number of pieces each triplet gets is $3 : 4 : 5$. If the Thompson triplets are ravenous and will eat as many pieces as possible, how many pieces will not be eaten by the Thompson triplets?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

35. **(2020 KMMC 8 P11)**
Two fair six-sided dice are rolled. What is the probability that the square of the sum of the numbers facing up on the dice is divisible by 8?

$\textbf{(A)}\ \frac{1}{6} \qquad \textbf{(B)}\ \frac{2}{9} \qquad \textbf{(C)}\ \frac{1}{4} \qquad \textbf{(D)}\ \frac{5}{18} \qquad \textbf{(E)}\ \frac{1}{3}$

36. **(2020 KMMC 8 P14)**
If Karate travels at 78 miles per hour for 100 minutes going from his house to the hospital, how many miles per hour would he need to travel in 65 minutes to travel from the hospital back to his house?

$\textbf{(A)}\ 100 \qquad \textbf{(B)}\ 110 \qquad \textbf{(C)}\ 120 \qquad \textbf{(D)}\ 130 \qquad \textbf{(E)}\ 140$

37. **(2020 KMMC 8 P15)**
Karate has a bag of marbles, where 3 of them are white, and the rest are black. He draws 4 marbles from the bag at random, all at once. If the probability of drawing 2 white marbles and 2 black marbles is equal to the probability of drawing 1 white marble and 3 black marbles, then how many marbles were in the bag at the start?

$\textbf{(A)}\ 6 \qquad \textbf{(B)}\ 8 \qquad \textbf{(C)}\ 9 \qquad \textbf{(D)}\ 10 \qquad \textbf{(E)}\ 13$

> **Remark.** This problem was originally proposed to the 2021 Spring DMC 10.

38. **(2020 KMMC 8 P16)**
How many ordered pairs of positive integers (x, y) satisfy

$$x^4 = y^2 + 8?$$

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 8$

> **Remark.** A previous problem was written by the AoPS user **GammaZero**, who used a somewhat similar equation in his problem.

39. **(2020 KMMC 8 P17)**
How many permutations of the word $KARATE$ are there such that the two A's are not next to each other?

$\textbf{(A)}\ 60 \qquad \textbf{(B)}\ 120 \qquad \textbf{(C)}\ 180 \qquad \textbf{(D)}\ 240 \qquad \textbf{(E)}\ 300$

> **Remark.** This problem was originally proposed to the 2021 Spring DMC 10.

40. **(2020 KMMC 8 P18)**
Karate has n pieces of candy. Karate realizes that there are an odd number of values of k for which he can split the pieces of candy into k different groups such that each group has an equal number of pieces of candy. If n is between 5 and 500, inclusive, then how many values of n are possible?

(A) 17 **(B)** 18 **(C)** 19 **(D)** 20 **(E)** 21

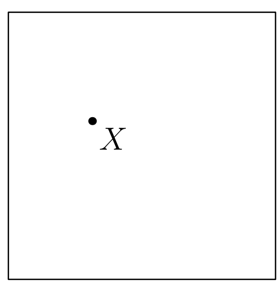
> **Remark.** This problem was inspired by a past AMC 8 problem in 2015, where one had to find the number of ways to arrange a class into rows such that each row had to have the same number of people.

41. **(2020 KMMC 8 P19)**
Karate has a whole number of cookies in his bag. If he had 5 more cookies than he currently does, he could give an equal number of cookies to 9 different people with none left over. If he had 2 fewer cookies than he currently does, he could give an equal number of cookies to 8 different people with none left over. Let N be the smallest possible number of cookies in Karate's bag. What is the sum of the digits of N?

(A) 11 **(B)** 12 **(C)** 13 **(D)** 14 **(E)** 15

> **Remark.** This problem was originally proposed to the 2021 Spring DMC 10.

42. **(2020 KMMC 8 P20)**
A point X is randomly chosen from the interior of a square with side length 2. What is the probability that X is within 1 unit from the midpoints of at least two sides of the square?



(A) $\frac{1}{2}$ **(B)** $\frac{\pi-1}{4}$ **(C)** $\frac{\pi-2}{2}$ **(D)** $\frac{\pi+2}{8}$ **(E)** $\frac{2\pi-1}{8}$

> **Remark.** This problem was originally proposed to the 2021 Spring DMC 10.

43. **(2020 KMMC 8 P21)**
There exists a positive real number x such that

$$x^2 + 4x - 2020 = 0.$$

What is the sum of the digits of the nearest integer to x?

(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

44. **(2020 KMMC 8 P23)**
If the number $8^a \cdot 9^b$ has 7800 positive integer divisors, where a and b are positive integers, what is the smallest possible value of a?

$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 8 \qquad \textbf{(D)}\ 13 \qquad \textbf{(E)}\ 17$

45. **(2020 KMMC 8 P24)**
Karate writes the first 10 positive perfect squares on a whiteboard. He then uses as many of the digits that he wrote as possible to create a multiple of 9. For example, with the digits 9, 9, 8, 2, and 1, he can create the number 9189. How many digits does Karate use?

$\textbf{(A)}\ 14 \qquad \textbf{(B)}\ 15 \qquad \textbf{(C)}\ 16 \qquad \textbf{(D)}\ 17 \qquad \textbf{(E)}\ 18$

> **Remark.** This problem was originally proposed to Season 2 OTSS for the never-released TMC 8.

46. **(2020 Fall LMT Team Round Division A P1/B P9)**
Ben writes the string

$$\underbrace{111\ldots11}_{\text{2020 digits}}$$

on a blank piece of paper. Next, in between every two consecutive digits, he inserts either a plus sign $(+)$ or a multiplication sign $(\times)$. He then computes the expression using standard order of operations. Find the number of possible distinct values that Ben could have as a result.

47. **(2020 Fall LMT Team Round Division A P2/B P6)**
1001 marbles are drawn at random and without replacement from a jar of 2020 red marbles and n blue marbles. Find the smallest positive integer n such that the probability that there are more blue marbles chosen than red marbles is strictly greater than $\frac{1}{2}$.

> **Remark.** The original problem had 13 marbles drawn from a jar of 20 red marbles and n blue marbles, which is something that the AoPS user **GammaZero** will likely understand.

48. **(2020 Fall LMT Team Round Division A P10/B P18)**
Define a sequence $\{a_n\}_{n\geq 1}$ recursively by $a_1 = 1$, $a_2 = 2$, and for all integers $n \geq 2$, $a_{n+1} = (n+1)^{a_n}$. Determine the number of integers k between 2 and 2020, inclusive, such that $k+1$ divides $a_k - 1$.

49. **(2020 Fall LMT Team Round Division A P13)**
Find the number of integers n from 1 to 2020 inclusive such that there exists a multiple of n that consists of only 5's.

> **Remark.** This problem was originally written by the AoPS user **GammaZero**, but I worked with him to improve the problem, namely by fixing technical issues in his original solution.

50. **(2020 Fall LMT Guts Round P6)**
The number 2021 can be written as the sum of 2021 consecutive integers. What is the largest term in the sequence of 2021 consecutive integers?

> **Remark.** This was the first problem that I wrote for the 2020 Fall LMT.

51. **(2020 Fall LMT Guts Round P27)**
A list consists of all positive integers from 1 to 2020, inclusive, with each integer appearing exactly once. Define a move as the process of choosing four numbers from the current list and replacing them with the numbers $1, 2, 3, 4$. If the expected number of moves before the list contains exactly two 4's can be expressed as $\frac{a}{b}$ for relatively prime positive integers, evaluate $a + b$.

> **Remark.** This problem was originally proposed to the 2020 Spring LMT, with the AoPS user **richy** improving my first draft of the problem. However, only the math team captains were able to propose problems for that contest, so they ended up moving this problem to the 2020 Fall LMT.

52. **(Season 2 TMC 10 P5)**
15 students are to be randomly split into 5 groups of 3 to work on a project. Alice, Bob, and Cooper are three of the students. Given that Alice and Bob are in the same group, what is the probability that Cooper is not in that group?

(A) $\frac{4}{7}$ **(B)** $\frac{3}{4}$ **(C)** $\frac{4}{5}$ **(D)** $\frac{8}{9}$ **(E)** $\frac{12}{13}$

> **Remark.** This problem was originally proposed to the never-released TMC 8.

53. **(Season 2 TMC 10 P10/12 P7)**
In a regular hexagon with side length 2, three of the sides are chosen at random. Next, the midpoints of each of the chosen sides are drawn. What is the probability that the triangle formed by the three midpoints has a perimeter which is an integer?

(A) $\frac{1}{10}$ **(B)** $\frac{1}{5}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{2}{5}$ **(E)** $\frac{1}{2}$

54. **(Season 2 TMC 10 P12/12 P9)**
A group of people are in a room. It is given that 5 people have a pet dog, 6 people have a pet cat, 8 people have a pet fish, and 3 people have no pets. If no one has more than two pets, and no one has more than one of the same type of pet, what is the smallest possible number of people in the room?

(A) 10 **(B)** 13 **(C)** 14 **(D)** 16 **(E)** 19

> **Remark.** This problem was originally proposed to the 2021 Spring DMC 10.

55. **(Season 2 TMC 12 P15)**
For how many positive integers $n \leq 15$ does there exist a positive integer k such that

$$\lfloor \log_2 k \rfloor + \lfloor \log_3 k \rfloor + \lfloor \log_4 k \rfloor + \cdots + \lfloor \log_8 k \rfloor = n?$$

(Here, $\lfloor r \rfloor$ denotes the largest integer less than or equal to r for all real numbers r.)

(A) 8 **(B)** 9 **(C)** 12 **(D)** 13 **(E)** 15

56. **(Season 2 TMC 12 P17)**
How many distinct cubic polynomials $P(x)$ with all integer coefficients and leading coefficient 1 exist such that $P(0) = 3$, $|P(1)| < 12$, and $P(x)$ has three (not necessarily real or distinct) roots whose squares sum to 34?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

Remark. This problem was originally proposed to the Season 1 TMC 12B.

Problems (2021)

1. **(Season 2 OTIE P2)**
Let $\mathcal{A}$ and $\mathcal{B}$ be two sets. Suppose that $\mathcal{A}$ contains a distinct elements and $\mathcal{B}$ contains b distinct elements, where a and b are positive integers. For some positive integer n, if there exist 2021 distinct elements belonging to at least one of $\mathcal{A}$ and $\mathcal{B}$, and there exist n distinct elements belonging to both $\mathcal{A}$ and $\mathcal{B}$, then the number of possible ordered pairs (a, b) is $2n$. Find n.

2. **(Season 2 OTIE P6)**
Let $\mathcal{S}$ be the set of all positive integers less than and relatively prime to 49. Call a subset of $\mathcal{S}$ with 15 distinct numbers *great* if it can be divided into 3 pairwise disjoint groups of 5 numbers such that no two numbers in the same group leave the same remainder when divided by 7, and the product of the numbers in each group leaves a unique remainder when divided by 7. Let n be the number of great subsets of $\mathcal{S}$. Find the sum of the (not necessarily distinct) primes in the prime factorization of n.

3. **(Season 2 OTIE P7)**
Let a and b be positive real numbers such that $\log_a b = \log_{ab} a^2$, $17ab = 60b + 1$, and $a \neq b$. The difference between the largest and smallest possible values of ab can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

4. **(Season 2 OTIE P8)**
Find the sum of the three least positive integers that cannot be written as
$$\frac{a!}{b!} + \frac{c!}{d!} + \frac{e!}{f!}$$
for positive integers a, b, c, d, e, f less than or equal to 5.

5. **(Season 2 OTIE P9)**
A jar contains five slips labeled from 1 to 5, inclusive. In each turn, Kevin takes two different slips out of the jar at random. If Kevin selects slips with the numbers a and b, the numbers a and b are replaced with the numbers 0 and $a + b$, and both slips are put back in the jar. Kevin stops once he writes the number 12 on a slip or takes three turns. The probability that the number 12 has been written once Kevin stops is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

6. **(Season 2 OTIE P10)**
Consider the polynomial
$$P(x) = x^{21} - 364x^{20} + Q(x),$$
where $Q(x)$ is some polynomial of degree at most 19. If the roots of $P(x)$ are all integers and $P(21) = 2021$, find the remainder when $P(23)$ is divided by 1000.

7. **(2021 Spring KMMC 10 P4)**
At a buffet, Karate eats sushi, shrimp tacos, lo mein, and chicken tacos in some order. If he eats sushi sometime before he eats shrimp tacos, but eats lo mein sometime after he eats chicken tacos, in how many possible orders could he have eaten them?

(A) 3 **(B)** 6 **(C)** 12 **(D)** 15 **(E)** 24

8. **(2021 Spring KMMC 10 P9)**
There exist positive integers n which satisfy at least half of the following conditions:

- n is not a prime number.
- $n+1$ is equal to a perfect square.
- $n+2$ is a prime number.
- $n+3$ is equal to one more than a perfect square.

What is the sum of all n from 1 to 10, inclusive?

(A) 4 **(B)** 11 **(C)** 17 **(D)** 20 **(E)** 21

> **Remark.** This is probably my least favorite problem I have written that was on a released math contest. While I believe that this problem could have been a pretty good first ten problem, I was too lazy to tweak it so that it was less bashy and annoying than it currently is.

9. **(2021 Spring KMMC 10 P16)**
What is the smallest possible value of

$$\left|\frac{x}{3}-20\right|+\left|\frac{x}{2}-10\right|+\left|\frac{x}{3}+10\right|$$

over all real numbers x?

(A) 20 **(B)** 25 **(C)** 30 **(D)** 35 **(E)** 40

> **Remark.** Notice the similarity to the first problem in the 2019 section? Yeah, I was pretty worried that people would catch on, so I decided not to use this problem in the 2021 Spring DMC 10. However, I was willing to put pretty much whatever we wanted in the KMMC 10, hence why this problem is on here.

10. **(2021 Spring KMMC 10 P21)**
Triangle ABC has $AB=8$, $BC=6$, $AC=11$. Let points D and E trisect side $\overline{BC}$ such that D is closer to B than C. Let F be the intersection of $\overline{AC}$ and the bisector of $\angle ABC$. Let X be the intersection of $\overline{AD}$ and $\overline{BF}$, and let Y be the intersection of $\overline{AE}$ and $\overline{BF}$. What is the ratio of the area of $\triangle AYF$ to the area of $\triangle AXB$?

(A) $\frac{25}{63}$ **(B)** $\frac{14}{33}$ **(C)** $\frac{16}{35}$ **(D)** $\frac{26}{55}$ **(E)** $\frac{10}{21}$

> **Remark.** This problem was proposed to the 2021 Spring DMC 10, but I ultimately did not use this problem because we had too much geometry, and the AoPS user **richy** was concerned that the problem was a bit too standard. However, I was willing to put pretty much whatever we wanted in the KMMC 10, hence why this problem is on here.

11. **(2021 Spring KMMC 10 P22)**
Four people are sitting evenly spaced at a circular table. At once, each person chooses to sit at the seat to their left, the seat to their right, or their current seat, with each seat having a one-third chance of being chosen. If two or more people sit at the same seat, the people who chose that seat leave the table. The people who did not leave sit at their chosen seats. What is the probability that exactly two people are left sitting?

(A) $\frac{1}{3}$ **(B)** $\frac{4}{9}$ **(C)** $\frac{13}{27}$ **(D)** $\frac{16}{27}$ **(E)** $\frac{56}{81}$

> **Remark.** This problem was in storage for quite a while. This problem was actually proposed in late 2019 as a proposal to the 2019 Fall LMT, and it almost actually got used in the theme round. However, it was later shut down due to complaints from certain math team members about the problem being too annoying and/or bashy, especially for a middle school competition (which, frankly, I disagree with).
>
> After that unfortunate incident, I proposed this same problem to OTSS (Spring), where it was suggested that this problem would appear as OTIE P6 or P7, as dealing with the casework required a good deal of precision. However, it was not selected to appear on the final draft, as we already had a lot of combinatorics problems.
>
> Finally, I moved this problem to DMC as a possible candidate for the 2021 Spring DMC 10. However, I grew paranoid due to this problem having been seen by many people up to this point. I did not want to start controversy, especially as this was my first released mock contest. But at long last, on February 2, 2021, this problem finally appeared on a released math contest, mock or real.

12. **(2021 Spring KMMC 10 P24)**
In triangle ABC with $AB = 13$, $BC = 14$, $AC = 15$, let O be the center of its circumcircle. Let P and Q be the feet of the perpendiculars from O to sides $\overline{AB}$ and $\overline{AC}$, respectively, and let E and F be the feet of the perpendiculars from B and C to line AO, respectively. What is $PE^2 + QF^2$?

(A) 98.5 **(B)** 101.5 **(C)** 104.5 **(D)** 107.5 **(E)** 110.5

> **Remark.** This problem was originally written by the AoPS user **i3435**, but I came in and kind of butchered the problem by having a really underwhelming solution. My bad!

13. **(2021 DIME P4)**
There are 7 balls in a jar, numbered from 1 to 7, inclusive. First, Richard takes a balls from the jar at once, where a is an integer between 1 and 6, inclusive. Next, Janelle takes b of the remaining balls from the jar at once, where b is an integer between 1 and the number of balls left, inclusive. Finally, Tai takes all of the remaining balls from the jar at once, if any are left. Find the remainder when the number of possible ways for this to occur is divided by 1000, if it matters who gets which ball.

> **Remark.** This problem was created with the AoPS user **firebolt360**.

14. **(2021 DIME P10)**
There exist complex numbers $z_1, z_2, \ldots, z_{10}$ which satisfy

$$|z_k i^k + z_{k+1} i^{k+1}| = |z_{k+1} i^k + z_k i^{k+1}|$$

for all integers $1 \leq k \leq 9$, where $i = \sqrt{-1}$. If $|z_1| = 9$, $|z_2| = 29$, and for all integers $3 \leq n \leq 10$, $|z_n| = |z_{n-1} + z_{n-2}|$, find the minimum value of $|z_1| + |z_2| + \cdots + |z_{10}|$.

> **Remark.** This problem was originally proposed to the Season 2 OTIE.

15. **(2021 Spring LMT Team Round Division A P11/B P17)**
In $\triangle ABC$ with $\angle BAC = 60^\circ$ and circumcircle ω, the angle bisector of $\angle BAC$ intersects side $\overline{BC}$ at point D, and line AD is extended past D to a point A'. Let points E and F be the feet of the perpendiculars of A' onto lines AB and AC, respectively. Suppose that ω is tangent to line EF at a point P between E and F such that $\frac{EP}{FP} = \frac{1}{2}$. Given that $EF = 6$, the area of $\triangle ABC$ can be written as $\frac{m\sqrt{n}}{p}$, where m and p are relatively prime positive integers, and n is a positive integer not divisible by the square of any prime. Find $m + n + p$.

> **Remark.** The original version of the problem said that A' was the reflection of A across D, when in reality, this is not the case.

16. **(2021 Spring LMT Team Round Division A P25/B P26)**
Chandler the Octopus is making a concoction to create the perfect ink. He adds 1.2 grams of melanin, 4.2 grams of enzymes, and 6.6 grams of polysaccharides. But Chandler accidentally added n grams of an extra ingredient to the concoction, Chemical X, to create glue. Given that Chemical X contains none of the three aforementioned ingredients, and the percentages of melanin, enzymes, and polysaccharides in the final concoction are all integers, find the sum of all possible positive integer values of n.

> **Remark.** The original problem was themed to the Powerpuff Girls, hence the mention of Chemical X. However, the LMT team decided to change the theme to Chandler the Octopus in order to match the actual LMT themes.

17. **(2021 Spring LMT Guts Round P22)**
A sequence $a_1, a_2, a_3, \ldots$ of positive integers is defined such that $a_1 = 4$, and for each integer $k \geq 2$,

$$2(a_{k-1} + a_k + a_{k+1}) = a_k a_{k-1} + 8.$$

Given that $a_6 = 488$, find $a_2 + a_3 + a_4 + a_5$.

18. **(2021 Fall DMC 10A P1)**
The sum of the first five positive integers and the sum of the first six positive integers are multiplied. What is the resulting product?

(A) 315 **(B)** 335 **(C)** 355 **(D)** 375 **(E)** 395

> **Remark.** This problem was originally proposed to the Season 2 TMC 10.

19. **(2021 Fall DMC 10A P13/11 P11)**
Let ω be the inscribed circle of a rhombus $ABCD$ with side length 4 and $\angle DAB = 60^\circ$. There exist two distinct lines which are parallel to line BD and tangent to ω. Given that the lines intersect sides $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$ at points P, Q, R, and S, respectively, what is the area of quadrilateral $PQRS$?

$\textbf{(A)}\ 2\sqrt{3} \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 3\sqrt{3} \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ 4\sqrt{3}$

> **Remark.** This problem was originally written by the AoPS user **AT2005**, but I modified the problem slightly. I believe that I could have done more with my modifications, though, because I believe that this problem was a bit standard.

20. **(2021 Fall DMC 10A P14/11 P12)**
Alice goes cherry picking in a forest. For each tree Alice sees, she either picks one cherry or three cherries from the tree and puts them in her basket. Additionally, after every five trees Alice picks from, she finds an extra cherry on the ground and puts it in her basket. At the end, Alice has 45 cherries in her basket. If the smallest possible number of trees Alice could have picked from is n, what is the sum of the digits of n?

$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 5 \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 7 \qquad \textbf{(E)}\ 8$

21. **(2021 Fall DMC 10A P15/11 P13)**
Each of 6 distinct positive integers is placed at each of 6 equally spaced points on the circumference of a circle. If the numbers on every two adjacent points are relatively prime, and the product of the numbers on every two diametrically opposite points is divisible by 3, what is the least possible sum of the 6 integers?

$\textbf{(A)}\ 12 \qquad \textbf{(B)}\ 25 \qquad \textbf{(C)}\ 26 \qquad \textbf{(D)}\ 29 \qquad \textbf{(E)}\ 32$

> **Remark.** This problem was originally written by the AoPS user **AT2005**, but I modified the problem by adding more conditions to make the problem more tricky.

22. **(2021 Fall DMC 10A P17/11 P15)**
Let $\triangle ABC$ have $AB = 20$, $AC = 21$, and a right angle at A. Let I be the center of the inscribed circle of $\triangle ABC$. Let point D be the reflection of point B over the line parallel to AB passing through I, and let point E be the reflection of point C over the line parallel to AC passing though I. What is the value of DE^2?

$\textbf{(A)}\ 145 \qquad \textbf{(B)}\ 149 \qquad \textbf{(C)}\ 153 \qquad \textbf{(D)}\ 157 \qquad \textbf{(E)}\ 161$

> **Remark.** This problem was originally written by the AoPS user **ApraTrip**, where it was a complicated AIME problem, but I came in and kind of butchered the problem by oversimplifying it. My bad!

23. **(2021 Fall DMC 10A P20/11 P17)**
Ann rolls two fair six-sided dice. If the sum of the numbers she rolled is at least 7, she rolls the dice again (and does not roll after that). Otherwise, she does not. What is the probability she rolls a 5 on at least one of the dice, on at least one of the rolls?

(A) $\frac{1}{3}$ **(B)** $\frac{13}{36}$ **(C)** $\frac{29}{72}$ **(D)** $\frac{11}{27}$ **(E)** $\frac{4}{9}$

> **Remark.** This problem had two instances where a piece of information was omitted: the first was "on at least one of the dice," and the second was "on at least one of the rolls."

24. **(2021 Fall DMC 10A P21/11 P18)**
In equilateral $\triangle ABC$, let points D and E be on lines AB and AC, respectively, both on the opposite side of line BC as A. If $CE = DE$, and the circumcircle of $\triangle CDE$ is tangent to line AB at D, what is the degree measure of $\angle CDE$?

(A) 70 **(B)** 72 **(C)** 75 **(D)** 80 **(E)** 84

> **Remark.** A previous problem was written by the AoPS user **i3435**, who used something with an equilateral triangle and length chasing. I experimented to create my own problem idea, and magically found this pretty interesting angle chasing problem with an unconventional answer. (No, I will not spoil the answer in this document.)

25. **(2021 Fall DMC 11 P20)**
Richard has four identical balls labeled 1, and two identical balls labeled -1. He randomly places each ball into one of six different bins, where he is allowed to place multiple balls in the same bin. What is the probability that the sum of the numbers of the balls in each bin is nonnegative? (A bin with no balls in it has sum 0.)

(A) $\frac{61}{441}$ **(B)** $\frac{23}{147}$ **(C)** $\frac{1}{6}$ **(D)** $\frac{3}{14}$ **(E)** $\frac{33}{98}$

26. **(2021 Fall DMC 10A P23/11 P21)**
In pentagon $ABCDE$, where all interior angles have a positive degree measure less than 180°, let M be the midpoint of side $\overline{DE}$. It is given that line BM splits $ABCDE$ into two isosceles trapezoids $ABME$ and $CDMB$ such that each one contains exactly three sides of equal length. If $AE = 3$ and $DE = 26$, what is the area of $ABCDE$?

(A) 216 **(B)** 234 **(C)** 288 **(D)** 312 **(E)** 330

> **Remark.** This problem idea was originally written by me for the 2021 Spring LMT, but it was too difficult and had too many things to keep track of. When I later proposed this problem to DMC, the AoPS user **jayseemath** suggested that the problem could give some side lengths and ask to find the area, and just like that, this problem came to be.

27. **(2021 Fall DMC 11 P23)**
An acute $\triangle ABC$ has $BC = 30$, $\angle BAC = 60^\circ$, $AC > AB$, and circumcircle ω with center O. The line tangent to ω at A intersects line BC at a point D. It is given that the line passing through O, parallel to line BC, intersects line AD at a point P such that $AP : DP = 4 : 3$. The length AD can be written as $m\sqrt{n}$, where m and n are positive integers, and n is not divisible by the square of any prime. What is $m + n$?

(A) 19 **(B)** 22 **(C)** 25 **(D)** 28 **(E)** 31

> **Remark.** I really wish that I had saved this problem for the second DIME.

28. **(2021 Fall DMC 11 P24)**
Let S be the set of all real numbers a for which there exists a positive real number b such that $\lfloor ab^2 \rfloor = 1$, $\lfloor a^2b^2 \rfloor = 2$, and $\lfloor a^3b^2 \rfloor = 6$, where $\lfloor r \rfloor$ denotes the greatest integer less than or equal to a real number r. As a approaches the greatest lower bound A of S, the set of all possible values of b approaches a set with a single element B. What is the value of $A^4 + B^4$?

(A) $\frac{31}{3}$ **(B)** $\frac{539}{48}$ **(C)** 13 **(D)** $\frac{265}{16}$ **(E)** $\frac{67}{4}$

> **Remark.** I really wish that I had saved this problem for the second DIME. Also, this problem had a correction two years after the test's release. At the time of writing this problem, I did not know that the greatest lower bound is a property of a set. The original problem statement supposes that the greatest lower bound pertains to a itself and not the set of all possible values of a.

29. **(2021 Fall DMC 10A P25/11 P25)**
Ryan has an infinite supply of slips and a spinner with letters O, S, and T, where each letter is equally likely to be spun. Each minute, Ryan spins the spinner randomly, writes on a blank slip the letter he spun, and puts it in a pile. Ryan continues until he has written all 3 letters at least once, at which point he stops. What is the probability that after he stops, he can form the words $OTSS$ and $TOST$ using 4 distinct slips from the pile? (Ryan may reuse slips he used for one word in forming the other.)

(A) $\frac{7}{54}$ **(B)** $\frac{13}{72}$ **(C)** $\frac{2}{9}$ **(D)** $\frac{8}{27}$ **(E)** $\frac{1}{3}$

30. **(Season 3 TIME P4)**
Find the number of positive integers $n \leq 1000$ such that

$$n(n+1)(n+\tfrac{1}{2})(n+\tfrac{1}{3})(n+\tfrac{1}{4})$$

is an integer.

> **Remark.** This problem was created with the AoPS user **Aathreyakadambi**.

31. **(Season 3 TIME P5)**
Let $P(x) = x^2 + ax + b$ be a quadratic with not necessarily distinct real roots r and s, where a and b are positive integers. If the quadratic $Q(x) = x^2 + 2ax + 3b$ has real roots r and $t \neq s$, find the maximum value of $P(1) + Q(1)$ less than 1000.

> **Remark.** I wrote this problem in around 10 minutes.

32. **(Season 3 TIME P7)**
During fencing practice, six people split into three pairs to spar. Two more people join afterwards. The eight people then randomly rearrange themselves into four pairs to spar. The probability that no one spars with someone that they previously sparred with is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

> **Remark.** This problem was created with the AoPS user **ivyzheng**.

33. **(Season 3 TIME P10)**
In triangle ABC with $AB = 26$, $BC = 28$, and $AC = 30$, let O and $\overline{AD}$ be the center and a diameter of the circumcircle of $\triangle ABC$, respectively. Two distinct lines pass through O, are parallel to $\overline{AB}$ and $\overline{AC}$, respectively, and meet side $\overline{BC}$ at points M and N, respectively. Let lines DM and DN meet the circumcircle of $\triangle ABC$ at points P and Q, respectively, both distinct from D. Find the area of $APDQ$.

> **Remark.** This problem was created with the AoPS user **NJOY**.

34. **(2021 Fall DMC 10B P1)**
What is the value of
$$2^0 \times 2^1 + 2^0 \times 2^2?$$

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

35. **(2021 Fall DMC 10B P2)**
How many single-digit positive integers n are there such that $2n$ is a perfect square?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

36. **(2021 Fall DMC 10B P4)**
What is the smallest positive integer n such that $n! + 1$ is not divisible by any integer between 2 and 9, inclusive?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

37. **(2021 Fall DMC 10B P6)**
For what values of k does the equation

$$k^2x + 2 = 4x + k$$

have no real solutions x?

(A) -2 **(B)** 0 **(C)** 2 **(D)** -2 and 0 **(E)** -2 and 2

38. **(2021 Fall DMC 10B P7)**
Justin has three weightless boxes and four pebbles, each of which has a weight of either 3, 4, or 5 ounces. He puts each pebble in one of the boxes such that each box has at least one pebble in it. If the weights of the boxes form an increasing arithmetic progression, what is the largest possible weight of the heaviest box, in ounces?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

39. **(2021 Fall DMC 10B P8)**
At Test Academy, there are four classes, one on each of the four floors of the building. For each class, the class which is one floor above it has twice as many students and half the average grade of that class. If the average grade of all four classes combined is 20, what is the average grade of the class on the bottom floor?

(A) 75 **(B)** 80 **(C)** 85 **(D)** 90 **(E)** 95

> **Remark.** This problem was created with the AoPS user **pog**.

40. **(2021 Fall DMC 10B P10)**
How many ordered triples of integers (a, b, c) are there such that the product

$$(a - 2020)(2b - 2021)(3c - 2022)$$

is positive and has exactly three positive divisors?

(A) 3 **(B)** 9 **(C)** 12 **(D)** 24 **(E)** infinitely many

> **Remark.** This problem was created with the AoPS user **pog**.

41. **(2021 Fall DMC 10B P13)**
Two functions f and g, in that order, are said to be *rivals* if there does not exist a real number x such that $f(x) = g(f(x))$. If f and g are linear, non-constant, and rivals, which of the following sets contains all possible values that $g(1)$ can never take?

(A) $\{-1\}$ **(B)** $\{0\}$ **(C)** $\{1\}$ **(D)** $\{-1, 1\}$ **(E)** the empty set

> **Remark.** The AoPS user **pog** wanted a problem whose title was "Rent a $g(x)f(x)$," so I made this problem. You're welcome.

42. **(2021 Fall DMC 10B P14)**
Rectangle $ABCD$ has $AB = 6$ and $BC = 4$. A circle passes through A and B and intersects side $\overline{CD}$ at two points which trisect the side. What is the area of the circle?

(A) 6π **(B)** 7π **(C)** 8π **(D)** 9π **(E)** 10π

43. **(2021 Fall DMC 10B P15)**
Let x be a positive real number such that

$$\frac{1}{x - \frac{1}{x}} = \sqrt{x^2 + \frac{x^4}{4}}.$$

What is the value of x^2?

(A) $4 - 2\sqrt{2}$ **(B)** $\sqrt{2}$ **(C)** $2\sqrt{2} - 1$ **(D)** 2 **(E)** $\sqrt{2} + 1$

> **Remark.** This problem was created with the AoPS user **HrishiP**.

44. **(2021 Fall DMC 10B P17)**
There exists a sequence $a_1, a_2, \ldots, a_6$ of positive integers such that for every term in the sequence, there exists another term in the sequence which is equal to that term. How many possible values of the product $a_1 a_2 \cdots a_6$ less than 1000 are there?

(A) 36 **(B)** 37 **(C)** 38 **(D)** 39 **(E)** 40

45. **(2021 Fall DMC 10B P18)**
In trapezoid $ABCD$ with $\overline{AB} \parallel \overline{CD}$, $AB = 4$, and $AD = BC = 5$, let the angle bisector of $\angle ADC$ intersect the diagonal $\overline{AC}$ at a point P. If line BP intersects the side $\overline{CD}$ at a point Q such that $CQ = 8$, what is the area of trapezoid $ABCD$?

(A) 24 **(B)** 28 **(C)** 32 **(D)** 36 **(E)** 40

> **Remark.** This problem was one of many contestants' favorite problems on the test.

46. **(2021 Fall DMC 10B P19)**
Six red balls and six blue balls are each numbered from 1 to 6. How many ways are there to form six pairs of one red ball and one blue ball such that the product of the two numbers on the balls in every pair is divisible by at least one of 2 and 3?

(A) 288 **(B)** 360 **(C)** 432 **(D)** 504 **(E)** 576

47. **(2021 Fall DMC 10B P20)**
At a motel, there are 15 rooms in a row. A visitor may rent 1 room for 5 dollars, or 2 adjacent rooms for 4 dollars each. At most 1 visitor may rent a given room at a time, and no 2 visitors may rent rooms adjacent to each other. If the leftmost and rightmost rooms must be rented, what is the largest dollar amount that the motel can earn?

(A) 40 **(B)** 41 **(C)** 42 **(D)** 43 **(E)** 44

48. **(2021 Fall DMC 10B P21)**
A convex quadrilateral $ABCD$ has $\angle ADC = \angle BAC = 90^\circ$ and side lengths $AB = 6$, $BC = 9$, and $CD = 5$. Let M be the midpoint of diagonal $\overline{BD}$. What is MC^2?

(A) 26 **(B)** 27 **(C)** 28 **(D)** 29 **(E)** 30

> **Remark.** Admittedly, the intended solution (which is featured on the official solutions booklet) is pretty out-of-the-blue.

49. **(2021 Fall DMC 10B P22)**
Bill and Ben each have 2 fair coins. Each minute, both Bill and Ben flip all their coins at the same time, if they have any. If a coin lands heads, then the other person gets that coin. If a coin lands tails, then that coin stays with the same person. What is the probability that after exactly 3 minutes, they each end up with 2 coins?

(A) $\frac{1}{4}$ **(B)** $\frac{5}{16}$ **(C)** $\frac{3}{8}$ **(D)** $\frac{7}{16}$ **(E)** $\frac{1}{2}$

> **Remark.** This problem was created with the AoPS user **HrishiP**.

50. **(2021 Fall DMC 10B P23)**
What is the sum of the digits of the smallest positive integer n such that

$$\sqrt{5n-1} - \sqrt{5n-2} + \sqrt{5n-3} - \sqrt{5n-4}$$

is less than 0.05?

(A) 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

51. **(2021 Fall DMC 10B P24)**
In $\triangle ABC$ with $AB = 3$ and $AC = 6$, let D be the intersection of the angle bisector of $\angle BAC$ and $\overline{BC}$, and let M be the midpoint of $\overline{AC}$. Let the circumcircle of $\triangle DMC$ intersect line AD again at P, distinct from D. If $DM = 2$, what is PC^2?

(A) $\frac{72}{5}$ **(B)** $\frac{78}{5}$ **(C)** $\frac{84}{5}$ **(D)** 18 **(E)** $\frac{96}{5}$

52. **(2021 Fall DMC 10C P1)**
What is the value of
$$4^1 - 3^2 + 2^3 - 1^4?$$

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

53. **(2021 Fall DMC 10C P3)**
Bill writes all odd perfect squares from 1 to 100, inclusive, and Jill writes all even perfect squares from 1 to 100, inclusive. Who writes more digits, and by how many?

(A) Bill, 1 **(B)** Bill, 2 **(C)** Jill, 1 **(D)** Jill, 2 **(E)** neither

> **Remark.** Admittedly, this problem was kind of last-minute.

54. **(2021 Fall DMC 10C P4)**
Given a right triangle with legs of lengths 5 and 6, a square is drawn with one side as its hypotenuse such that the triangle is completely inside the square. What is the area of the region inside the square but outside the triangle?

(A) 46 **(B)** 47 **(C)** 48 **(D)** 49 **(E)** 50

55. **(2021 Fall DMC 10C P6)**
John is playing a game with 6 levels, each with 5 stages. After the third stage of each of the first five levels, John may choose whether or not to skip the remaining stages in the level and start at the first stage of the next level. If John finished the whole game, how many possible combinations of stages could John have played through?

(A) 5 **(B)** 10 **(C)** 16 **(D)** 30 **(E)** 32

> **Remark.** Admittedly, this problem was not very good in my opinion (and in some contestants' opinions).

56. **(2021 Fall DMC 10C P7)**
What is the sum of all positive real numbers a such that the equation $x^2 + ax - 12 = 0$ has two distinct integer solutions x?

(A) 6 **(B)** 12 **(C)** 14 **(D)** 16 **(E)** 22

57. **(2021 Fall DMC 10C P8)**
Daniel has to walk one mile to complete his gym homework. He decides to split his path into quarters, where after each quarter, he randomly chooses to turn 90° clockwise or counterclockwise with equal probability. If Daniel walks in a straight line each quarter, what is the probability that he will end up where he started after walking the mile?

$\textbf{(A)}\ \frac{1}{8} \qquad \textbf{(B)}\ \frac{1}{4} \qquad \textbf{(C)}\ \frac{1}{2} \qquad \textbf{(D)}\ \frac{5}{8} \qquad \textbf{(E)}\ \frac{3}{4}$

58. **(2021 Fall DMC 10C P9)**
Let a and b be positive integers. If a is divisible by 2 but not 3, and b is divisible by 3 but not 2, what is the greatest possible three-digit value of $a+b$?

$\textbf{(A)}\ 995 \qquad \textbf{(B)}\ 996 \qquad \textbf{(C)}\ 997 \qquad \textbf{(D)}\ 998 \qquad \textbf{(E)}\ 999$

> **Remark.** This is my favorite problem in the first ten of either the DMC 10B or DMC 10C (for some reason).

59. **(2021 Fall DMC 10C P10)**
How many orderings of the six numbers 1, 1, 2, 2, 3, and 6 are there such that the sum of the first three numbers is twice the sum of the last three numbers?

$\textbf{(A)}\ 9 \qquad \textbf{(B)}\ 18 \qquad \textbf{(C)}\ 27 \qquad \textbf{(D)}\ 36 \qquad \textbf{(E)}\ 72$

60. **(2021 Fall DMC 10C P12)**
In a plane, eight rays emanate from a point P such that every two adjacent rays form an acute angle with measure 45°. Next, a line segment with a finite length is drawn in the plane. If the line segment intersects exactly n of the rays, what is the sum of all possible values of n? (If the line segment passes through P, then $n=8$.)

$\textbf{(A)}\ 13 \qquad \textbf{(B)}\ 14 \qquad \textbf{(C)}\ 17 \qquad \textbf{(D)}\ 18 \qquad \textbf{(E)}\ 23$

61. **(2021 Fall DMC 10C P14)**
Draw two identical non-intersecting circles, a line tangent to both circles at distinct points A and B, where the circles are on the same side of the line, and a line tangent to both circles at distinct points C and D, where the circles are on opposite sides of the line. The lines intersect at point P. If $AB=11$ and $CD=5$, what is $AP\cdot BP$?

$\textbf{(A)}\ 20 \qquad \textbf{(B)}\ 22 \qquad \textbf{(C)}\ 24 \qquad \textbf{(D)}\ 26 \qquad \textbf{(E)}\ 28$

> **Remark.** Although a bit hard for a P14, this is probably one of my favorite problems on the test.

62. **(2021 Fall DMC 10C P19)**
A car moves such that if there are n people in it, it moves at a constant rate of 4^n miles per hour. At noon, the car has 1 person in it and starts moving. After every mile, another person instantaneously gets in the car. How many people are in the car when the average speed the car has moved since noon reaches 17 miles per hour?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

63. **(2021 Fall DMC 10C P21)**
Richard thinks of a positive integer n and writes the base ten representations of $n!$ and $(n+1)!$ on a board. He then erases the zeroes to the right of the last nonzero digit of each number (if any exist), resulting in two numbers a and b. If one of a and b is 4 times the other, what is the sum of all possible values of n less than 1000?

(A) 315 **(B)** 441 **(C)** 656 **(D)** 714 **(E)** 819

> **Remark.** This problem was very easy to get wrong.

64. **(2021 Fall DMC 10C P22)**
In the xy-plane are perpendicular lines $y = ax + d$ and $y = bx + c$, where a, b, c, and d are real numbers in a geometric progression in that order. If the two lines and the line $y = \frac{3}{2}x$ pass through a common point, what is the least possible value of $a + b + c + d$?

(A) $\frac{3}{2}$ **(B)** $\frac{51}{32}$ **(C)** $\frac{13}{8}$ **(D)** $\frac{111}{64}$ **(E)** $\frac{7}{4}$

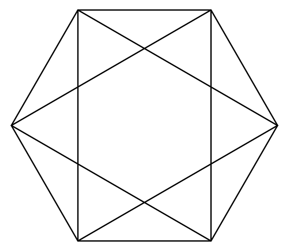
> **Remark.** For some reason, this is one of my favorite problems on either the DMC 10B or DMC 10C.

65. **(2021 Fall DMC 10C P23)**
There are 15 people in a room, where everyone shakes hands with a positive number of other people in the room exactly once. If exactly 6 people shook 1 hand, exactly 5 people shook between 2 and 4 hands, inclusive, exactly 1 person shook 8 hands, and exactly 1 person shook 14 hands, what is the least possible total number of handshakes?

(A) 24 **(B)** 25 **(C)** 26 **(D)** 27 **(E)** 28

> **Remark.** For some reason, this was some people's favorite problem on the test and other people's least favorite.

66. **(2021 Fall DMC 10C P25)**
Each of the six vertices of the regular hexagon shown below is labeled with either a 1 or a 2. Some diagonals of the hexagon are drawn, and each of the six points of intersection is labeled with either a 2, a 3, or a 4. In how many ways can the 12 points be labeled such that for every drawn diagonal of the hexagon, the sum of the numbers on its two endpoints is *not* equal to either of the numbers on the two points of intersection of the diagonal? Rotations and reflections are considered distinct.



(A) 502 **(B)** 514 **(C)** 526 **(D)** 538 **(E)** 550

> **Remark.** The main reason why this problem made it on here is because we wanted to do a callback to the first DMC 10 in 2020, where its P25 was also polygonal casework. However, we thought that that problem was far too unreasonable to solve under timed conditions, especially for an AMC 10. Thus, we made the casework more reasonable here, albeit still quite involved. Ultimately, we thought that this problem would serve as a fitting finale of sorts to the DMC 10 series in 2021. With that being said, we still think the bashy-ness may still be much for even a P25 on an AMC 10, so we apologize in advance... but I just couldn't resist. :P

67. **(2021 GAMC P1)**
What is the area of the largest circle which can fit entirely within the interior of a semicircle with diameter 24?

(A) 24π **(B)** 36π **(C)** 48π **(D)** 72π **(E)** 144π

> **Remark.** Originally, I wrote this problem where it was the largest semicircle that can fit entirely within the interior of a circle. However, the AoPS user **depsilon0** found this to be trolly, so we decided to switch the roles of the semicircle and the circle.

68. **(2021 ADMC Individual Round 1 P5)**
A trapezoid $ABCD$ has $\overline{AB} \parallel \overline{CD}$, $AD = BD = CD$, and $\angle ABD = 54^\circ$. If $\angle ACB = a^\circ$, find a.

> **Remark.** This problem was originally written by the AoPS user **bissue**, but I tweaked the problem.

69. **(2021 ADMC Individual Round 2 P5)**
Janelle sees a right triangle with all integer side lengths and computes the sum of the squares of the side lengths. Afterwards, she writes down her sum in both base-2 and base-3. To her surprise, Janelle discovers that her two expressions have the same second-to-last digit but a different last digit. Find the sum of all possible remainders when Janelle's sum is divided by 36.

> **Remark.** This problem was originally proposed to the 2021 Fall DMC 10C.

70. **(2021 ADMC Team Round P6)**
Let $\triangle ABC$ have $AB = 5$, $BC = 9$, and $AC = 13$. Let D and E be points on line BC, with the points D, B, E, and C lying in that order, such that $AC = CD$ and $AB = BE$. Let the angle bisector of $\angle ACB$ intersect $\overline{AB}$ and $\overline{AD}$ at points F and G, respectively. The ratio of the area of $BEFG$ to the area of $\triangle ABC$ is equal to $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $100m + n$.

71. **(2021 KMMC 8A P1)**
Karate's daily training starts at 6:10 PM and ends at 7:45 PM. If Karate takes a 20-minute break in the middle of his training, for how many minutes does he train?

(A) 65 **(B)** 75 **(C)** 85 **(D)** 95 **(E)** 105

72. **(2021 KMMC 8A P4)**
Karate has a recipe for hot chocolate which requires 2 grams of cocoa powder and 5 grams of milk. After adding 5 grams of milk, Karate accidentally adds 3 grams of cocoa powder, so he adds additional milk in the same proportion as the recipe to balance the cocoa powder out. How many grams of additional milk does he add?

(A) 1 **(B)** 2.5 **(C)** 5 **(D)** 7.5 **(E)** 10

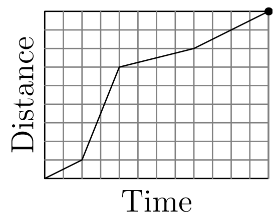
> **Remark.** This problem was created with the AoPS user **pog**.

73. **(2021 KMMC 8A P6)**
Karate has a bag of sweets consisting of 30% pieces of chocolate, 45% pieces of toffee, and the rest pieces of caramel. After giving half of his caramel to his wife, he has 15 pieces of caramel left. How many pieces of toffee does he have?

(A) 27 **(B)** 36 **(C)** 54 **(D)** 60 **(E)** 120

> **Remark.** This problem was created with the AoPS user **pog**.

74. **(2021 KMMC 8A P8)**
One day, Karate hiked through a forest for two hours. The graph below shows his hike, indicating the general time and distance hiked. Which of the following represents a time m in minutes after the hike started where Karate is moving the fastest?



(A) 22 **(B)** 44 **(C)** 66 **(D)** 88 **(E)** 110

75. **(2021 KMMC 8A P9)**
Given that $a = 0.78$ and $b = 78$, which of the following is equal to 7800?

(A) $\frac{a}{b^2}$ **(B)** $\frac{a^2}{b}$ **(C)** $\frac{b^2}{a}$ **(D)** $\frac{b^3}{a}$ **(E)** $\frac{b^3}{a^2}$

76. **(2021 KMMC 8A P13)**
Karate has 3 coins, each of which is either a penny (a 1-cent coin), a nickel (a 5-cent coin), or a dime (a 10-cent coin). If Karate trades one of his coins for a quarter (a 25-cent coin), the total value of his 3 coins will then be twice the total value of his original 3 coins. What is the least possible total value in cents of his original 3 coins?

(A) 14 **(B)** 15 **(C)** 17 **(D)** 20 **(E)** 24

Remark. Admittedly, this problem had terrible answer choices.

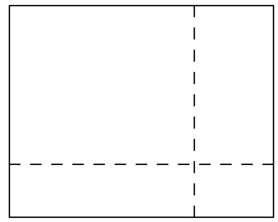
77. **(2021 KMMC 8A P14)**
What is the value of the expression

$$\left(\frac{2^2}{1+3}-\frac{4^2}{3+5}\right)+\left(\frac{6^2}{5+7}-\frac{8^2}{7+9}\right)+\cdots+\left(\frac{98^2}{97+99}-\frac{100^2}{99+101}\right)?$$

(A) -100 **(B)** -50 **(C)** -25 **(D)** -10 **(E)** -5

Remark. I came up with this problem during English class.

78. **(2021 KMMC 8A P18)**
A rectangle is cut using only vertical or horizontal cuts, like the cuts shown below. What is the smallest possible number of cuts needed to cut the rectangle into 36 pieces?



(A) 5 **(B)** 10 **(C)** 12 **(D)** 18 **(E)** 35

79. **(2021 KMMC 8A P20)**
Three boys named Karate, Judo, and Naruto, as well as two girls named Haruka and Ayaka, sit in a straight line in a randomly chosen order. What is the probability that exactly one boy and one girl are sitting in between Karate and Judo?

(A) $\frac{1}{15}$ **(B)** $\frac{1}{10}$ **(C)** $\frac{2}{15}$ **(D)** $\frac{1}{6}$ **(E)** $\frac{1}{5}$

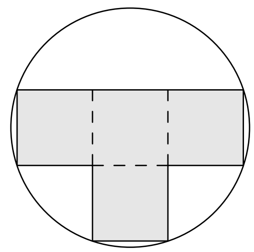
80. **(2021 KMMC 8A P21)**
Karate has five blocks in a row, as shown below, where each letter represents a number, and not all five numbers are equal.

[A] [B] [C] [D] [E]

Karate notices that the sums of the numbers on any three consecutive blocks are equal. In which of the following arrangements of the five blocks is it necessarily true that the sums of the numbers on any three consecutive blocks are equal?

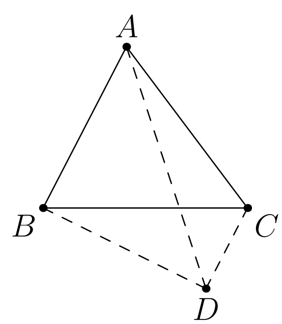
(A) [A] [C] [D] [B] [E] **(B)** [B] [A] [C] [D] [E] **(C)** [C] [A] [B] [E] [D]
(D) [D] [E] [C] [A] [B] **(E)** [E] [A] [B] [D] [C]

81. **(2021 KMMC 8A P22)**
A T-shaped block is inscribed in a circle, as shown below. The T-shaped block is made up of four squares. Given that the area of the circle is 78π, which of the following is closest to the area of the T-shaped block?



(A) 105 **(B)** 110 **(C)** 115 **(D)** 120 **(E)** 125

82. **(2021 KMMC 8A P25)**
In triangle ABC, let D be the point on the opposite side of line BC as A such that $\overline{AB} \parallel \overline{CD}$. Given that $AB = BD = 8$, $BC = 9$, and $CD = 4$, what is AD^2?



(A) 126 **(B)** 130 **(C)** 134 **(D)** 138 **(E)** 142

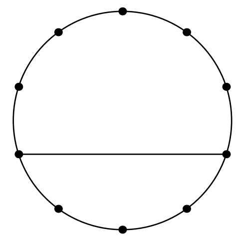
83. **(2021 Fall KMMC 10 P2)**
What is the smallest positive integer which can be expressed as the sum of a positive perfect square and a distinct positive perfect cube?

(A) 2 **(B)** 3 **(C)** 5 **(D)** 9 **(E)** 10

84. **(2021 Fall KMMC 10 P4)**
A positive integer n is divisible by 5 but not 4. For which of the following values will adding it to n never result in a sum that is divisible by 20?

(A) 45 **(B)** 50 **(C)** 55 **(D)** 60 **(E)** 65

85. **(2021 Fall KMMC 10 P5)**
Ten points are equally spaced on the circumference of a circle, where two of the points are connected by a line segment, as shown below. Karate wants to choose two of the eight other points and draw a line segment connecting them so that this line segment does not intersect the other line segment. In how many ways can Karate do this?



(A) 11 **(B)** 12 **(C)** 13 **(D)** 14 **(E)** 15

86. **(2021 Fall KMMC 10 P8)**
Let n be the number of factors of 3 that the product

$$(1+2+3)(4+5+6)(7+8+9)\cdots(94+95+96)(97+98+99)$$

contains. What is the sum of the digits of n?

(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

87. **(2021 Fall KMMC 10 P9)**
A rectangle $ABCD$ has $AB = 8$ and $BC = 4$. Points P and Q lie on sides $\overline{AB}$ and $\overline{BC}$, respectively, such that $AP = CQ$ and the area of $\triangle BPQ$ is 6. What is PQ^2?

(A) 32 **(B)** 34 **(C)** 36 **(D)** 38 **(E)** 40

88. **(2021 Fall KMMC 10 P12)**
Karate has some shirts, pairs of pants, and socks. An outfit consists of one shirt, one pair of pants, and two socks. Karate can currently wear 60 possible different outfits, but if he were to get either four more shirts or four more socks, then Karate could wear 180 possible different outfits. How many pairs of pants does Karate have? (The socks are distinguishable, and the order in which he wears the socks does not matter.)

(A) 2 **(B)** 5 **(C)** 6 **(D)** 8 **(E)** 15

89. **(2021 Fall KMMC 10 P14)**
In the xy-plane, the point $(24, 7)$ is reflected over the line $y = ax$ and then shifted up b units to the point $(20, 21)$, where a and b are positive real numbers. What is $a \cdot b$?

(A) 3 **(B)** 4 **(C)** 6 **(D)** 8 **(E)** 12

90. **(2021 Fall KMMC 10 P15)**
Karate repeatedly rolls two fair six-sided dice. After every roll, Karate writes the sum of the numbers on the two dice. Karate stops once he writes the number 11. What is the probability that Karate writes the number 5 at least once before stopping?

(A) $\frac{1}{3}$ **(B)** $\frac{5}{12}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{2}{3}$ **(E)** $\frac{3}{4}$

91. **(2021 Fall KMMC 10 P16)**
A pyramid $ABCDE$ has rectangular base $ABCD$ with $AB = 8$ and $BC = 6$ and apex E located 8 units above $ABCD$. A plane parallel to $\triangle ACE$ hits segments $\overline{AB}$, $\overline{BC}$, and $\overline{BE}$ at F, G, and H, respectively. If $FG = \frac{15}{2}$, what is the volume of $DFGH$?

(A) 27 **(B)** 36 **(C)** 45 **(D)** 48 **(E)** 60

92. **(2021 Fall KMMC 10 P18)**
An isosceles trapezoid $ABCD$ with $\overline{AB} \parallel \overline{CD}$, $AB = AD = BC = 3$, and $CD = 7$ is inscribed in a circle. A point E on the circle satisfies $\overline{AC} \perp \overline{BE}$. What is DE^2?

(A) 24 **(B)** 26 **(C)** 28 **(D)** 30 **(E)** 32

93. **(2021 Fall KMMC 10 P19)**
Let a, b, c, and d be positive integers. If

$$\frac{a!}{b!} + \frac{c!}{d!} = \frac{2}{5},$$

what is the largest possible value of $a + b + c + d$?

(A) 10 **(B)** 18 **(C)** 26 **(D)** 34 **(E)** 42

94. **(2021 Fall KMMC 10 P20)**
Let $\triangle ABC$ with $AB = 14$ be inscribed in a circle with center O. Let P be a point on side $\overline{BC}$, and let line AP intersect the circle at a point D, distinct from A. If $\triangle ABC$ is acute and $ABDO$ is a rhombus, what is the largest possible integer value of BP^2?

(A) 33 **(B)** 48 **(C)** 65 **(D)** 78 **(E)** 95

95. **(2021 Fall KMMC 10 P21)**
Consider the polynomial

$$P(x) = (x - a)(x - b)(x - c)(x - d),$$

where a, b, c, and d are fixed positive real numbers less than 1. What is the maximum possible number of distinct real numbers r that can satisfy $|P(r)| = 1$?

(A) 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 8

96. **(2021 Fall KMMC 10 P23)**
Karate has the string $AABBABBB$. Judo is told the string has 8 letters, each an A or a B, but not the string itself or how many of each letter are in it. Judo is told that each move, Karate will pick the ith letter of the existing string, that letter and every letter to its right will switch from A to B and vice versa, and Judo will learn what i is and how many of each letter are in the new string. If Karate moves optimally and Judo reasons well, what is the fewest number of moves for Judo to know the string?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

97. **(2021 Fall KMMC 10 P24)**
In right $\triangle ABC$ with $AB = 8$, $AC = 6$, and a right angle at A, let M be the midpoint of side $\overline{BC}$, and let D be the reflection of C over line AM. Line segments $\overline{AC}$ and $\overline{DM}$ are extended to meet at a point E. What is the length of $\overline{CE}$?

(A) $\frac{32}{5}$ **(B)** $\frac{120}{17}$ **(C)** $\frac{50}{7}$ **(D)** $\frac{95}{13}$ **(E)** $\frac{84}{11}$

98. **(2021 Fall KMMC 10 P25)**
What is the largest positive integer k for which there exist positive integers a and b, where $a < b$, such that $a + b > 48$ and $1 < \sqrt[3]{a^3 + kb^2} - a < 2$?

(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

99. **(2022 DIME P4)**
Given a regular hexagon $ABCDEF$, let point P be the intersection of lines BC and DE, and let point Q be the intersection of lines AP and CD. If the area of $\triangle QEP$ is equal to 72, find the area of regular hexagon $ABCDEF$.

100. **(2022 DIME P6)**
In $\triangle ABC$ with $AC > AB$, let D be the foot of the altitude from A to side $\overline{BC}$, and let M be the midpoint of side $\overline{AC}$. Let lines AB and DM intersect at a point E. If $AC = 8$, $AE = 5$, and $EM = 6$, find the square of the area of $\triangle ABC$.

101. **(2021 KMMC 8B P1/9 P1)**
Karate spent \$1.75 on candy, \$2.15 on soda, and \$4.25 on popcorn. If Karate pays with only 1-dollar bills, what is the least possible amount of change he could get back?

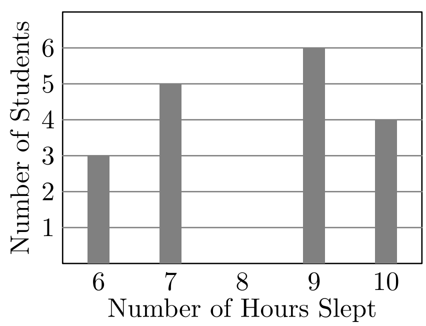
(A) \$0.15 **(B)** \$0.40 **(C)** \$0.55 **(D)** \$0.60 **(E)** \$0.85

102. **(2021 KMMC 8B P4)**
Aki and Ayaka take a total of eight pictures in which at least one of them appears. If Aki appears in five of the pictures, and Ayaka appears in seven of the pictures, in how many of the pictures do they both appear?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

> **Remark.** This problem was created with the AoPS user **pandabearcat**.

103. **(2021 KMMC 8B P10/9 P8)**
In a survey, Karate asked the students of his third grade class how many hours of sleep each student got last night. The bar graph below shows the results of Karate's survey. However, the bar representing 8 hours has been mysteriously erased. If the median number of hours slept is 8.5, how many students slept for 8 hours?



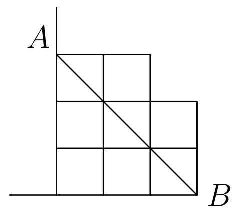
(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

> **Remark.** This problem was created with the AoPS user **pandabearcat**.

104. **(2021 KMMC 8B P14)**
Karate has some unread books in his room. Today, Karate plans to read 10 of his books. Tomorrow, Karate plans to read exactly half of his remaining books. The day after tomorrow, Karate plans to read 8 of his remaining books. After this, Karate will have read exactly three-quarters of his books. How many books does Karate have?

(A) 28 **(B)** 36 **(C)** 45 **(D)** 48 **(E)** 52

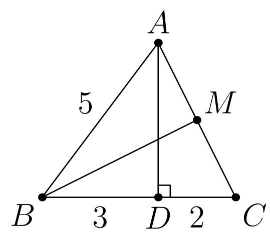
105. **(2021 KMMC 8B P15/9 P12)**
Karate pushes n unit squares to a wall. Then, he puts three unit squares on top and pushes them to the wall. Finally, he puts two unit squares on top and pushes them to the wall. The figure below shows the resulting shape for $n = 3$. If points A and B represent the top-left and bottom-right vertices of the shape, respectively, what is the largest value of n such that segment $\overline{AB}$ does not go outside of the shape?



(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

> **Remark.** Not gonna lie, pretty proud of this one.

106. **(2021 KMMC 8B P21/9 P18)**
In $\triangle ABC$, let D be the foot of the altitude from A to side $\overline{BC}$, and let M be the midpoint of side $\overline{AC}$. If $BD = 3$, $CD = 2$, and $AB = 5$, what is the length of $\overline{BM}$?



(A) 4 **(B)** $2\sqrt{5}$ **(C)** $2\sqrt{6}$ **(D)** $3\sqrt{3}$ **(E)** $4\sqrt{2}$

> **Remark.** This problem was created with the AoPS user **PhunsukhWangdu**.

Problems (2022)

1. **(2022 KMMC 2A P2)**
Which of these numbers has the largest tens digit?

(A) 32 **(B)** 78 **(C)** 123 **(D)** 756 **(E)** 1048

2. **(2022 KMMC 2A P3)**
How many letters in the word $KARATE$ are vowels?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

3. **(2022 KMMC 2A P4)**
Exactly two years ago, Karate was 16 years old. How many years old will Karate be exactly four years from now?

(A) 20 **(B)** 21 **(C)** 22 **(D)** 23 **(E)** 24

4. **(2022 KMMC 2A P8)**
In a race between 10 people, Karate finished first in the race, and Judo finished last in the race. If there were no ties in the race, how many people finished behind Karate but ahead of Judo?

(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

> **Remark.** This problem was created with the AoPS user **pog**.

5. **(2022 KMMC 2A P9)**
Karate has 58 pencils. Judo then gives him 40 pencils. After Judo gives Karate the pencils, Naruto then takes away 74 of Karate's total pencils. How many pencils does Karate have now?

(A) 20 **(B)** 24 **(C)** 28 **(D)** 32 **(E)** 36

> **Remark.** This problem was created with the AoPS user **pandabearcat**.

6. **(2022 KMMC 2A P10)**
Haruka is skip-counting by 4. She starts by saying the number 15, then the number 19, then the number 23, and so on. Which of these numbers will she eventually say?

(A) 30 **(B)** 31 **(C)** 32 **(D)** 33 **(E)** 34

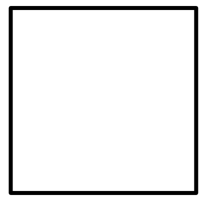
7. **(2022 KMMC 2A P13)**
For all numbers $\square$ and $\triangle$, the value of $\square \ominus \triangle$ is equal to $\triangle - \square$, where $-$ is the subtraction sign. What is the value of $3 \ominus 5$?

(A) -2 **(B)** -1 **(C)** 0 **(D)** 1 **(E)** 2

8. **(2022 KMMC 2A P14)**
If there are twelve inches in a foot, and there are three feet in a yard, how many inches long is a stick which is a yard and two feet long?

(A) 24 **(B)** 36 **(C)** 48 **(D)** 60 **(E)** 72

9. **(2022 KMMC 2A P16)**
Which of the following is **not** true about a square?



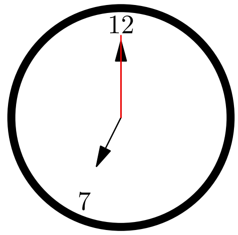
(A) Squares have four sides. **(B)** All sides of a square are equal.
(C) Squares are rectangles. **(D)** Squares have two lines of symmetry.
(E) Squares have more sides than triangles.

> **Remark.** This problem was created with the AoPS user **pog**.

10. **(2022 KMMC 2A P18)**
Karate has 20 sheets of paper. He wants to create packets by stapling either two sheets of paper or three sheets of paper together. If Karate makes seven packets with two sheets of paper each, how many packets with three sheets of paper each can Karate make with the sheets of paper left over?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

11. **(2022 KMMC 2A P20)**
At 7:00, all of the gears of an analog clock stopped working. Karate wants to display 7:30 on the clock. Which of the three hands of the analog clock does Karate have to move?



(A) hour hand only **(B)** minute hand only **(C)** seconds hand only
(D) both hour hand and minute hand **(E)** all three hands

> **Remark.** This problem was created with the AoPS user **pog**.

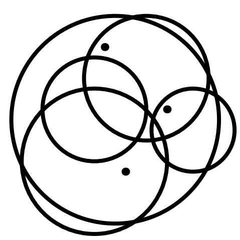
12. **(2022 KMMC 2A P21)**
Karate is taller than Judo but shorter than Haruka. Naruto is taller than Ayaka but shorter than Judo. Who is the shortest among the five people?

(A) Karate **(B)** Judo **(C)** Naruto **(D)** Haruka **(E)** Ayaka

13. **(2022 KMMC 2A P22)**
A recipe for one bowl of soup calls for 3 carrots, 2 cups of water, and 3 potatoes. If Karate has 9 carrots, 7 cups of water, and 8 potatoes, at most how many bowls of soup can he make?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

14. **(2022 KMMC 2A P23)**
How many of the circles have at least one dot inside of them?



(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

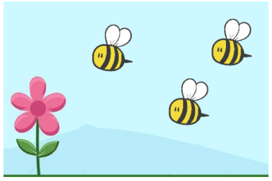
15. **(2022 KMMC 2A P24)**
Aki and Judo each have 10 grapes. Aki gives Judo 2 of his grapes. Then, Judo gives Aki 5 of his grapes. Finally, Aki eats 3 of his grapes. Afterwards, who has more grapes, and by how many?

(A) Aki, 1 **(B)** Aki, 3 **(C)** Judo, 1 **(D)** Judo, 3 **(E)** neither

16. **(2022 KMMC 2A P25)**
A group of people are standing in a row, including Karate and Judo. Karate is standing somewhere to the right of Judo. As well, there are 7 people standing to the left of Karate, and there are 5 people standing to the right of Judo. If there are 2 people standing in between Karate and Judo, how many people are standing in the row, including Karate and Judo?

(A) 10 **(B)** 11 **(C)** 12 **(D)** 13 **(E)** 14

17. **(2022 KMMC 2B P1)**
How many bees are in the picture?



(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

> **Remark.** This problem was created with the AoPS user **pandabearcat**.

18. **(2022 KMMC 2B P3)**
How many letters of the word $YUKI$ also appear in the word $KARATE$?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

> **Remark.** This problem was created with the AoPS user **pog**.

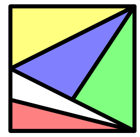
19. **(2022 KMMC 2B P4)**
Which of these numbers becomes larger when the digits of the number are put in backwards order (from right to left)?

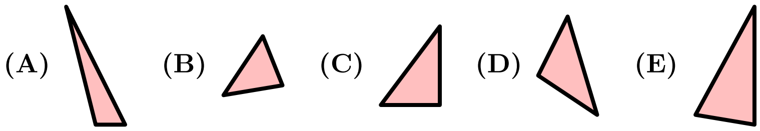
(A) 7 **(B)** 42 **(C)** 65 **(D)** 78 **(E)** 121

20. **(2022 KMMC 2B P5)**
Judo is grilling steak for dinner. He wants the steak to be grilled for at least five minutes, but no longer than seven minutes. Which of these choices is a good number of minutes for Judo to grill his steak?

(A) 2 **(B)** 4 **(C)** 6 **(D)** 8 **(E)** 10

21. **(2022 KMMC 2B P6)**
Haruka has an unfinished square puzzle with a red piece, a yellow piece, a blue piece, and a green piece. Haruka wants to complete her puzzle by using one pink piece. Which of these pink pieces should Haruka use? (Haruka can turn and flip over pieces, but she cannot break them apart.)





> **Remark.** All that Asymptote just to make one simple problem...

22. **(2022 KMMC 2B P7)**
Which of the following is true about the number 4?

(A) It is greater than 6. **(B)** It is less than 1. **(C)** It is equal to $1 + 3$.
(D) It is an odd number. **(E)** It is a two-digit number.

23. **(2022 KMMC 2B P9)**
Karate and Judo play a game where each round, either one player wins and the other player loses, or they tie. After seven rounds, Karate won three rounds, and two rounds ended in a tie. How many rounds did Judo lose?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

24. **(2022 KMMC 2B P10)**
If $1 + 2 + \square + \triangle = 10$, what is the value of $\square + \triangle$?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

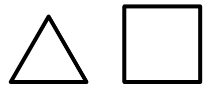
25. **(2022 KMMC 2B P11)**
What is the fifth smallest whole number which has at least one 1 as a digit?

(A) 12 **(B)** 13 **(C)** 14 **(D)** 15 **(E)** 16

26. **(2022 KMMC 2B P12)**
Ayaka has four empty bowls. For each bowl, she can put either one or two scoops of ice cream in it. If the four bowls have a combined total of seven scoops in them, how many of the four bowls have two scoops in them?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

27. **(2022 KMMC 2B P13)**
Aki has a triangle and a square where all of the triangle's and the square's sides have the same length. If Aki glues together a side of the triangle and a side of the square so that they exactly line up and the shapes do not overlap, he will form a new shape. How many sides will this new shape have?



(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

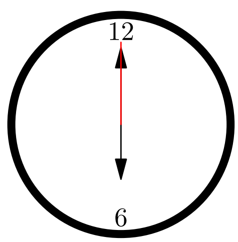
> **Remark.** This problem was created with the AoPS user **pandabearcat**.

28. **(2022 KMMC 2B P15)**
The composers Johann Sebastian Bach and Franz Joseph Haydn share the same birthday, where Bach was born in the year 1685, and Haydn was born in the year 1732. How many years before Haydn was born was Bach born?

(A) 41 **(B)** 43 **(C)** 45 **(D)** 47 **(E)** 49

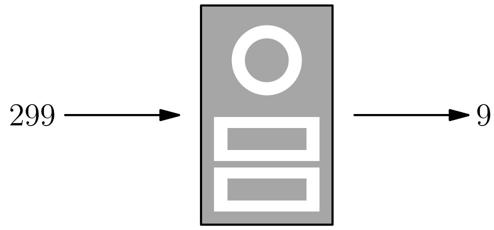
> **Remark.** This problem was created with the AoPS users **pandabearcat** and **pog**.

29. **(2022 KMMC 2B P16)**
Karate sees an analog clock showing the time 6:00. Karate will then look at the clock for the next 30 minutes. During those 30 minutes, how many times will Karate see the hour hand and the minute hand meet each other?



(A) 0 **(B)** 1 **(C)** 2 **(D)** 29 **(E)** 30

30. **(2022 KMMC 2B P17)**
A machine reads the digits of a number and outputs the largest digit that it read. (For example, putting the number 299 into the machine would output the digit 9.) When put into the machine, how many two-digit numbers would output the digit 2?



(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

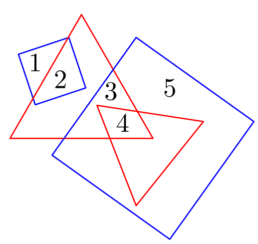
31. **(2022 KMMC 2B P18)**
Ayaka is skip-counting by threes and fives. She starts by saying the number 13, then the number 16, then the number 21, then the number 24, and so on, where she switches between going up by three and going up by five with each number. Which number will Ayaka **not** eventually say?

(A) 29 **(B)** 32 **(C)** 37 **(D)** 42 **(E)** 45

32. **(2022 KMMC 2B P19)**
Which of these numbers can be added to each of the numbers 17, 23, and 35 so that all three of the resulting numbers only have even digits?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

33. **(2022 KMMC 2B P20)**
How many of the five numbers are **not** inside of any red triangles?



(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

> **Remark.** This problem was created with the AoPS users **pandabearcat** and **pog**.

34. **(2022 KMMC 2B P21)**
In 40 minutes of nonstop work time, Karate can either draw two self-portraits or bake one pie. In 240 minutes of nonstop work time, Karate baked four pies. How many self-portraits did Karate draw during that time?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

35. **(2022 KMMC 2B P23)**
On the first Thursday of January, Judo learns that the next issue of his favorite comic series, *Karate the K-Pop Star*, will come out on the fourth Saturday of January. If the first Saturday of January was before the first Thursday of January, how many days will Judo have to wait before the issue comes out?

(A) 15 **(B)** 16 **(C)** 19 **(D)** 22 **(E)** 23

> **Remark.** This problem was created with the AoPS user **pandabearcat**.

36. **(2022 KMMC 2B P25)**
Karate tells Judo that his favorite whole number is greater than 7 and less than 12. Judo asks Karate, "Is your number odd?" Karate replies, "No, it is not." Judo can ask one more question to Karate. Which question should Judo ask in order to figure out Karate's favorite number without a doubt?

(A) Is your number less than 11? **(B)** Is your number greater than 10?
(C) Is your number even? **(D)** Does your number end with a 2?
(E) Does your number have two digits?

37. **(2022 KMJJIME P2)**
Seven years ago, Saya's age was half her brother's age. Saya is currently 15 years old. Find how many years old Saya's brother currently is.

> **Remark.** This problem was created with the AoPS user **pandabearcat**.

38. **(2022 KMJJIME P4)**
Judo makes a 17-inch straw by gluing some 3-inch straws and 4-inch straws together. Find how many total straws (3-inch and 4-inch) Judo used.

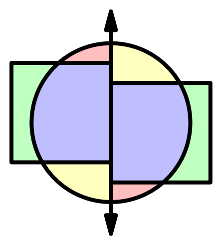
39. **(2022 KMJJIME P6)**
Karate has four different whole numbers, where three of the numbers are 10, 7, and 12, and the other number is unknown. If Karate can split his four numbers into two pairs so that the sums of the two numbers in both pairs are equal, find the greatest possible value of the unknown number.

40. **(2022 KMJJIME P10)**
Karate writes all of the whole numbers from 1 to 100 (including 1 and 100) on a sheet of paper. Then, Karate erases all numbers that are multiples of 3 from the paper. Then, Karate erases all numbers whose tens digit is equal to 2 from the paper. Find how many whole numbers are left on the paper.

41. **(2022 KMJJIME P11)**
Aki and Hanami each have a whole number from 1 to 20 (including 1 and 20). They only know that at least one of their numbers is odd. Aki says, "My number is less than 7, but it is greater than 4." Hanami says, "Then, I know that my number is at least three times as large as yours." Aki says, "Then, I know your number." Find the sum of Aki and Hanami's numbers.

> **Remark.** This problem was created with the AoPS user **pog**. I like this logic problem, and it is at an adequate difficulty.

42. **(2022 KMJJIME P12)**
Aki has two squares and a circle, where the area of the circle is twice the area of one square. He cuts the circle in half with a line and glues the squares on opposite sides of the line. In the picture, regions with the same color are identical. If the sum of the areas of 1 red, 1 yellow, 2 green, and 3 blue regions is $90\frac{3}{4}$ square inches, find the perimeter of one square, in inches.

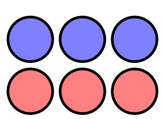


> **Remark.** A lot of Asymptote, but I think the problem is pretty good.

43. **(2022 KMJJIME P13)**
Karate chooses two different whole numbers and writes their sum, their positive difference, and their product on a board. If there is exactly one even number on the board, and that number is equal to 72, find the sum of all possible values of the smaller of Karate's two numbers.

> **Remark.** This problem was created with the AoPS user **pog**. I think this was a pretty neat idea.

44. **(2022 KMJJIME P14)**
Karate, Judo, Naruto, Haruka, Ayaka, and Saya each stand on one of the six circles in the picture so that each circle has exactly one person standing on it. Find how many ways they can stand so that Karate and Judo both stand on blue circles, and Naruto and Saya stand on circles with different colors.



> **Remark.** This problem was created with the AoPS user **pog**.

45. **(2022 KMJJIME P15)**
Karate has two whole numbers $\square$ and $\triangle$ such that

$$\square + \triangle = 2022 \quad \text{and} \quad 4 < \frac{\square}{\triangle} < 5.$$

Find the number of possible values of $\triangle$.

> **Remark.** This problem was created with the AoPS user **pog**.

46. **(berfoer P1)**
Consider the number 2022.

a. What is the sum of the digits of the number?
b. What is the closest perfect square to the number?
c. Let the perfect square found in (b) be N^2. What is the closest perfect square to N?
d. Is the fractional part of $\sqrt{N}$ or $\sqrt[4]{2022}$ greater? Explain.

47. **(berfoer P2)**
Two sports teams, the Planes and the Helicopters, play against each other in a 24-game series. In each game, either the Planes or the Helicopters win, and there are no ties. At the end of the 24 games, whichever team has won more games is the winner. Otherwise, the series ends in a tie. Suppose that there have been eight games so far, and the Planes have won five of them.

a. What percent of the first eight games did the Planes win?
b. How many of the remaining 16 games would the Planes need to win in order to beat the Helicopters in the series?
c. Is it possible for the Helicopters to win more games than the Planes after the first 10 games (given the results of the first eight games)? Explain.

Now, suppose that the series is split into quarters, where quarter 1 consists of the first six games, quarter 2 consists of the 7th to 12th games, and so on.

d. Is it possible for the Helicopters to have lost every game in quarter 1 (given the results of the first eight games)? Explain.
e. If the Planes beat the Helicopters in the series, what is the smallest possible number of games the Planes could have won in the quarter where they won the most games? Explain.

48. **(berfoer P3)**
There are some number of people in a room, where everyone is either male or female. A group of three people from the room is *commendable* if the group has at least one person of each gender. Suppose that two males, Ryan and Richard, and two females, Katherine and Katie, are in the room.

a. How many possible commendable groups can be formed with three people chosen from only Ryan, Richard, Katherine, and Katie? (The order in which the people are chosen does not matter.)
b. Can a group be formed with three people from these four that is **not** commendable? Explain.

Suppose that there are a total of 5 males in the room, including Ryan and Richard, and the total number of people in the room is $3N$, where N is a positive integer.

c. How many possible commendable groups of three people can be formed among the 5 males and Katherine and Katie if neither Ryan nor Richard are in it? (The order in which the people are chosen does not matter.)
d. If the people in the room can be split into N groups of 3 people such that every group is commendable, at least how many females must be in the room (including Katherine and Katie)?

Next, some more people join the room, so there are now 13 males and a total of $3M$ people in the room, where M is a positive integer.

e. If the people in the room can be split into M groups of 3 people such that every group is commendable, how many possible values of M are there, and what is the smallest possible value of M? Explain.
f. For the smallest value of M found in (e), suppose that the $3M$ people are split into M groups of 3 again. What is the largest possible number of groups that are **not** commendable that can be formed?

49. **(berfoer P4)**
Let $ABCD$ be an isosceles trapezoid with $\overline{AB} \parallel \overline{CD}$, $AB < CD$, and $AB + CD = AD$. Let P be the point on side $\overline{AD}$ such that $AB = AP$ and $DC = DP$. Suppose that $ABCD$ has perimeter 36.

a. If $\angle ABC = \theta$, what is the measure of $\angle ADC$ in terms of θ?
b. What is the length of $\overline{AD}$?

Let the perpendicular bisectors of $\overline{PB}$ and $\overline{PC}$ intersect at a point Q. Suppose that Q lies on side $\overline{BC}$.

c. What is the measure of $\angle AQD$? Explain.
d. What must the measure of $\angle BPC$ be? Explain.
e. What is the length of $\overline{PQ}$?
f. Is $\overline{PQ}$ parallel to $\overline{AB}$ and $\overline{CD}$? Explain.
g. Is the circle passing through P, B, and C tangent to line AD? Explain.
h. Let M be the midpoint of $\overline{PQ}$. Let $\overline{BM}$ and $\overline{AQ}$ intersect at a point X, and let $\overline{CM}$ and $\overline{DQ}$ intersect at a point Y. What is the length of $\overline{XY}$?

50. **(Pi P1)**
What is the smallest positive integer n such that the nearest integer to $n\pi$ is greater than $n\pi$?

51. **(Pi P5)**
In Ryan's really riveting report, he defines the *bicimal point* as the point separating a real number's integer and fractional parts in its binary (base-2) representation. For example, the binary number 10.1_2's bicimal point is between the 0 and the second 1. How many digits are there in the binary representation of the number π^2 before the bicimal point?

52. **(Pi P7)**
Ithaca is selling pies. Unfortunately, he forgot exactly how many pies he sold, so he relies on his memory. He remembers that he sold at least 31 pies between 1:01 and 2:00, exactly 41 pies between 2:01 and 3:00, and exactly 59 pies between 3:01 and 4:00. Furthermore, Ithaca sold as many pies between 1:01 and 2:30 as he did between 2:31 and 4:00. If Ithaca sold n pies between 2:01 and 2:30, how many possible values of n are there?

53. **(Pi P8)**
Here on Earth, we have a technique known as PIE, the Principle of Inclusion and Exclusion. However, on the planet of Pi, Pians, the inhabitants of Pi, do not believe in exclusion, so they have PI, the Principle of Inclusion. Thus, if there are 31 Pians who eat bacon for breakfast and 41 Pians who eat eggs for breakfast, then PI would state that $31 + 41 = 72$ of these Pians eat at least one of bacon and eggs, regardless of how many Pians eat both bacon and eggs. Now, if 15 more Pians were to arrive on Pi, all of whom eat both bacon and eggs, then the value which PIE would give (i.e. the actual value) when finding how many Pians eat at least one of bacon and eggs would be exactly two-thirds the value which PI would give. How many Pians were on Pi who eat both bacon and eggs before the 15 additional Pians arrived?

54. **(Pi P10)**
Gerolamo has distinct real number values π_1 and π_2 such that

$$\cos(\pi_1) + \cos(\pi_2) = \frac{6}{13} \quad \text{and} \quad \sin(\pi_1) + \sin(\pi_2) = \frac{8}{13}.$$

Let $(\cos(\pi_1) - \cos(\pi_2))^2 + (\sin(\pi_1) - \sin(\pi_2))^2 = \frac{m}{n}$, where m and n are relatively prime positive integers. What is $m + n$? (Note that π_1 and π_2 are not to be confused with radians!)

55. **(2022 Spring LMT Speed Round P22)**
Let $\triangle ABC$ be an isosceles triangle with $AB = AC$. Let M be the midpoint of side $\overline{AB}$. Suppose there exists a point X on the circle passing through points A, M, and C such that $BMCX$ is a parallelogram and M and X are on opposite sides of line BC. Let segments $\overline{AX}$ and $\overline{BC}$ intersect at a point Y. Given that $BY = 8$, find AY^2.

56. **(2022 DMC 10A P2/12A P2)**
What is the smallest positive integer n such that

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{n}$$

is less than 1?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

57. **(2022 DMC 10A P6/12A P5)**
There are 10 students in a classroom with 12 chairs. Before lunch, each student sat on a different chair. After lunch, each student randomly chose a chair to sit on. If no two students chose the same chair after lunch, what is the probability that every chair had been sat on at least once?

(A) $\frac{7}{24}$ **(B)** $\frac{11}{36}$ **(C)** $\frac{25}{66}$ **(D)** $\frac{14}{33}$ **(E)** $\frac{15}{22}$

> **Remark.** This problem was created with the AoPS user **pog**. Much like 2021 Fall DMC 10B P10, I took the original problem and made it much harder. This is probably misplaced for a P6/P5.

58. **(2022 DMC 10A P7/12A P6)**
Bo writes down all the divisors of 144 on a board. He then erases some of the divisors. If no two of the divisors left on the board have a product divisible by 18, what is the least number of divisors Bo could have erased?

(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

59. **(2022 DMC 10A P9/12A P7)**
Let a, b, and c be positive real numbers such that the ratio of a to bc is $1:3$, the ratio of b to ac is $1:12$, and the ratio of c to $a+b$ is $1:8$. What is $b+c$?

(A) 22 **(B)** 24 **(C)** 26 **(D)** 28 **(E)** 30

> **Remark.** This problem was created with the AoPS user **pog**.

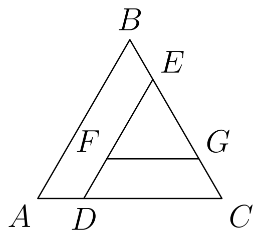
60. **(2022 DMC 10A P10)**
In how many ways can a non-empty subset A of $\{1,2,3,4\}$ and a non-empty subset B of $\{3,4,5,6\}$ be chosen so that A is a subset of B?

(A) 6 **(B)** 12 **(C)** 16 **(D)** 20 **(E)** 36

61. **(2022 DMC 10A P11/12A P9)**
In rectangle $ABCD$ with $AB=5$ and $BC=4$, let E be a point on side $\overline{CD}$. Given that segment $\overline{AE}$ bisects $\angle BED$, what is the length of $\overline{AE}$?

(A) $3\sqrt{2}$ **(B)** $2\sqrt{5}$ **(C)** $3\sqrt{3}$ **(D)** $2\sqrt{7}$ **(E)** $\sqrt{30}$

62. **(2022 DMC 10A P13)**
In equilateral $\triangle ABC$, let D and E be on $\overline{AC}$ and $\overline{BC}$, respectively, such that $CD = CE$, and let F and G be on $\overline{DE}$ and $\overline{CE}$, respectively, such that $EF = EG$. If the perimeters of $CDFG$ and $ABED$ are 17 and 22, respectively, and $AD = DF$, what is the perimeter of $\triangle EFG$?



(A) 12 **(B)** $12\frac{3}{4}$ **(C)** $13\frac{1}{2}$ **(D)** $14\frac{1}{4}$ **(E)** 15

> **Remark.** This problem was created with the AoPS user **pog**.

63. **(2022 DMC 10A P14)**
Ryan has 5 pieces of taffy and 6 pieces of gum, which he randomly distributes to three boys all at once. If each boy ends up with at least one piece of each sweet (taffy and gum), what is the probability that a boy ends up with more pieces of taffy but fewer pieces of gum than each of the other boys?

(A) $\frac{1}{20}$ **(B)** $\frac{1}{12}$ **(C)** $\frac{1}{10}$ **(D)** $\frac{1}{6}$ **(E)** $\frac{1}{5}$

> **Remark.** This problem was originally proposed to the 2021 ADMC.

64. **(2022 DMC 10A P17)**
In trapezoid $ABCD$ with $\overline{AB} \parallel \overline{CD}$, $AB = 3$, $CD = 7$, and $AD = BC$, let M be the midpoint of side $\overline{BC}$. If the circle with diameter $\overline{DM}$ is tangent to line AB, what is the length of the altitude from $\overline{AB}$ to $\overline{CD}$?

(A) $2\sqrt{3}$ **(B)** $\sqrt{15}$ **(C)** 4 **(D)** $3\sqrt{2}$ **(E)** $2\sqrt{5}$

65. **(2022 DMC 10A P19/12A P13)**
A positive integer $n > 1$ is called *toasty* if for all integers m with $1 \le m < n$, there exists a positive integer k such that

$$\frac{m}{n} = \frac{k}{k+12}.$$

How many toasty integers are there?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 6

66. **(2022 DMC 10A P20/12A P14)**
Let $\lfloor r \rfloor$ denote the greatest integer less than or equal to a real number r. Let N be the number of positive integers $n \leq 100$ such that

$$\lfloor (n+1)\pi \rfloor - \lfloor n\pi \rfloor = 4.$$

What is the sum of the digits of N?

(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

67. **(2022 DMC 12A P16)**
How many sequences of the first 8 positive integers $a_1, a_2, \ldots, a_8$ are there such that $a_{2i-1} < a_{2i}$ for all odd i, $a_{2i-1} > a_{2i}$ for all even i, and the even integers within the sequence are listed in increasing order?

(A) 42 **(B)** 66 **(C)** 78 **(D)** 102 **(E)** 108

> **Remark.** Too hard for P16.

68. **(2022 DMC 12A P17)**
Let $ABCD$ be a trapezoid with $\overline{AB} \parallel \overline{CD}$, $AB < CD$, and $AD = BC = 5$. Let M be the midpoint of side $\overline{AD}$. Given that $BM = 6$, and the area of $ABCD$ is as large as possible, what is $AB + CD$?

(A) 11 **(B)** $\frac{45}{4}$ **(C)** $\frac{35}{3}$ **(D)** 13 **(E)** $\frac{66}{5}$

69. **(2022 DMC 10A P21/12A P18)**
Each of the N students in Mr. Ji's class took a 10-question quiz with questions $1, 2, \ldots, 10$. Suppose for every (possibly empty) subset of $\{1, 2, \ldots, 10\}$, there exists a student who got exactly those questions correct, and for every $i = 0, 1, 2, \ldots, 10$, if a student got i questions correct, then of the students that got those same i questions correct (including that student), the fraction of them that got over i questions correct is $1 - 2^{i-10}$. If 3 students got a perfect score, what is the remainder when N is divided by 100?

(A) 24 **(B)** 32 **(C)** 48 **(D)** 64 **(E)** 72

> **Remark.** This problem was created with the AoPS user **HrishiP**.

70. **(2022 DMC 10A P22/12A P19)**
In isosceles $\triangle ABC$ with $AB = AC = 4$ and $BC = 2$, let point D, distinct from B, be on side $\overline{AB}$ such that $CD = 2$. The circle passing through B, C, and D intersects side $\overline{AC}$ and the line through C perpendicular to $\overline{AB}$ at points P and Q, respectively, both distinct from C. If PQ^2 is equal to $\frac{m}{n}$, where m and n are relatively prime positive integers, what is $m + n$?

(A) 45 **(B)** 46 **(C)** 63 **(D)** 64 **(E)** 71

> **Remark.** This problem was created with the AoPS user **HrishiP**.

71. **(2022 DMC 10A P23/12A P20)**
Let $ABCD$ be a rectangle with $AB > BC$. Let E be a point on side $\overline{AD}$, and let $CEFG$ be the rectangle where B is on side $\overline{FG}$. Let H be the point on side $\overline{CD}$ such that $\overline{BH} \perp \overline{CE}$. If $CH = 2$, $DE = 3$, and the area of $CEFG$ is 48, then $AC = m\sqrt{n}$, where m and n are positive integers, and n is not divisible by the square of any prime. What is $m + n$?

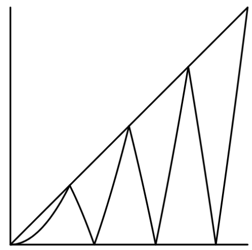
(A) 24 **(B)** 26 **(C)** 28 **(D)** 30 **(E)** 32

72. **(2022 DMC 10A P24/12A P21)**
How many ordered pairs of positive integers (m, n) satisfy the following?

"There are exactly m set(s) of 100 consecutive positive integers whose least element is less than 100 which contain exactly $m + 1$ multiples of n."

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

73. **(2022 DMC 10A P25/12A P23)**
In the xy-plane, a laser emanates from the origin with a path whose shape obeys $y = x^2$. Whenever the laser touches the line $y = x$, the path of the laser will reflect over the line parallel to the x-axis passing though where the laser last touched $y = x$, and whenever the laser touches the x-axis, the path of the laser will reflect over the x-axis. The graph below shows the path of the laser and its first 7 reflection points. If N denotes the sum of the squares of the x-coordinates of the first 20 points where the laser intersects the x-axis (excluding the origin), what is the sum of the digits of N?



(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

> **Remark.** This problem was created with the AoPS user **HrishiP**.

74. **(ANOTHER MOCK CONTEST :O P4)**
Karate and Judo are fighting in the parking lot. The moment a person gets hit 50 times, they lose the fight. If Karate has already hit Judo 37 times, and Judo has already hit Karate 40 times, what is the largest number of total additional hits possible without one person losing the fight after the last of these hits?

> **Remark.** This was probably originally written for the KMMC 2, though I don't remember which contest in particular.

75. **(ANOTHER MOCK CONTEST :O P5)**
In how many ways can 2022 be written as the sum of two nonnegative integers whose digits are only 0s and 1s? (The order in which the two integers are written does not matter.)

76. **(ANOTHER MOCK CONTEST :O P10)**
Let R, A, C, K, E, and T be distinct single-digit positive integers. If

$$A \cdot T = 20,$$
$$R \cdot E \cdot K \cdot T = 30,$$
$$R \cdot A \cdot C \cdot K \cdot E \cdot T = 720,$$

what is $A + C + T$?

> **Remark.** I recall that I proposed a very similar question to the 2019 Fall LMT.

77. **(ANOTHER MOCK CONTEST :O P13)**
How many ordered triples of (not necessarily distinct) primes (p, q, r) each less than 25 are there such that $p + q = r$?

78. **(ANOTHER MOCK CONTEST :O P23)**
Six distinct points lie in a straight line. First, Cindy draws a red line segment, choosing two of the six points to be the endpoints. Next, Sophia draws a blue line segment, choosing two of the six points to be the endpoints (these can include Cindy's points). Finally, Michael shades any points not on either line segment yellow. If any point on only the red segment is red, any point on only the blue segment is blue, and any point on both the red and blue line segments is purple, find the number of possible colorings of the points. (One coloring is red, purple, blue, blue, yellow, yellow.)

> **Remark.** This problem was originally proposed to the 2021 ADMC.

79. **(ANOTHER MOCK CONTEST :O P25)**
Let acute $\triangle ABC$ have $AB = 9$, $AC = 16$, and $\angle ACB = 30^\circ$. Let O be the center of the circumcircle of $\triangle ABC$, and let A' be the reflection of A over point O. Construct line ℓ which passes through O and is parallel to side $\overline{AB}$. Let P be the intersection of ℓ and the bisector of $\angle A'AB$. The perimeter of $ABPA'C$ can be written as $m\sqrt{n}+p$, where m, n, and p are positive integers, and n is not divisible by the square of any prime. Find $m+n+p$.

> **Remark.** This problem was originally proposed to the Season 2 OTIE.

80. **(2022 DMC 10B P2)**
When Amanda multiplies her favorite number by 3, subtracts the result from 14, and divides the result by 4, the resulting number will be Amanda's favorite number. What is Amanda's favorite number?

(A) -14 **(B)** -7 **(C)** 2 **(D)** 7 **(E)** 14

81. **(2022 DMC 10B P7/12B P5)**
Let a, b, and c be consecutive positive integers, and let p, q, and r also be consecutive positive integers, both not necessarily in order. Given that $a \cdot p = 161$ and $b \cdot q = 189$, what is $c \cdot r$?

(A) 128 **(B)** 144 **(C)** 160 **(D)** 176 **(E)** 192

82. **(2022 DMC 10B P8)**
How many ways can the six variables in the equation

$$a+b+c = d+e+f+2$$

be set equal to one of the numbers 1, 2, and 3 such that each of the three numbers is used by exactly two variables?

(A) 9 **(B)** 18 **(C)** 24 **(D)** 27 **(E)** 36

> **Remark.** This problem was originally proposed to mathleague.org.

83. **(2022 DMC 10B P9)**
Daniel is walking on a field, starting at his house. After walking 3 miles due north and 4 miles due west, Daniel arrives at the market. From the market, if Daniel walks n miles due east, Daniel will arrive at his school, which is the same distance from his house as it is from the market. What is n?

(A) $\frac{9}{4}$ **(B)** $\frac{21}{8}$ **(C)** 3 **(D)** $\frac{25}{8}$ **(E)** $\frac{25}{6}$

84. **(2022 DMC 10B P17/12B P12)**
A line passes through the point $A(5,4)$ and has slope -8. A second line passes through the point $B(1,6)$ and intersects the first line at a point C, equidistant from A and B. What is the slope of the second line?

(A) $\frac{1}{3}$ **(B)** $\frac{2}{5}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{6}{11}$ **(E)** $\frac{4}{7}$

85. **(2022 DMC 12B P13)**
Given that

$$\log_2 5 \cdot \log_3 6 \cdot \log_4 7 \cdot \log_5 8 = \log_2 a + \log_3 a,$$

what is the nearest integer to a?

(A) 15 **(B)** 17 **(C)** 19 **(D)** 21 **(E)** 23

> **Remark.** I wrote this problem on a whim, but feedback was generally quite positive.

86. **(2022 DMC 10B P18/12B P14)**
A rectangle has perimeter 36. The rectangle is split into three smaller rectangles with dimensions 9-by-4, 6-by-5, and m-by-n. What is $m+n$?

(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

87. **(2022 DMC 12B P15)**
Let $w = \frac{1}{2} + \frac{\sqrt{3}}{2}i$ and $z = -\frac{\sqrt{3}}{2} + \frac{1}{2}i$. How many ordered pairs of positive integers (a,b) each at most 6 are there such that $|3w^a + 4z^b| = 5$?

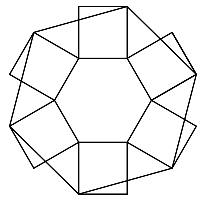
(A) 4 **(B)** 6 **(C)** 8 **(D)** 12 **(E)** 16

88. **(2022 DMC 10B P20/12B P16)**
There are 2022 members in a math tournament, where 999 members are girls, and the rest are boys. The members are split into 674 groups of 3. For every two members in a group, if at least one of them is a girl, they shake hands. Otherwise, they do not. A member is *handy* if they shake hands with both members in their group. Let N be the maximum number of handy members in the math tournament. What is the sum of the digits of N?

(A) 6 **(B)** 15 **(C)** 16 **(D)** 22 **(E)** 23

> **Remark.** My memory tells me that this problem was on an older (and no longer visible) version of the 2021 Fall DMC 10B, except this version has different numbers.

89. **(2022 DMC 10B P21/12B P17)**
In the figure below, six congruent rectangles are glued to each of the sides of a regular hexagon with side length 2, and six of the vertices of the rectangles are connected to form a regular hexagon with side length 4. The length of a side of one of the rectangles not equal to 2 can be written as $\sqrt{m} - \sqrt{n}$, where m and n are positive integers. What is $m + n$?



(A) 15 **(B)** 16 **(C)** 17 **(D)** 18 **(E)** 19

> **Remark.** Oops, pretty similar to 2008 AMC 10A P25.

90. **(2022 DMC 10B P23/12B P20)**
Let parallelogram $ABCD$ have $BC = 5$, $\angle ABC < 90°$, and $\angle ACB > 90°$. Let line AD and side $\overline{CD}$ intersect the circle passing though A, B, and C at $P \neq A$ and $Q \neq C$, respectively. If $CP = 10$ and $CQ = 4$, what is AP?

(A) $\frac{48}{7}$ **(B)** 7 **(C)** $\frac{36}{5}$ **(D)** $\frac{15}{2}$ **(E)** 8

91. **(2022 DMC 12B P25)**
Convex quadrilateral $ABCD$ has acute angles $\angle A$ and $\angle D$, obtuse angles $\angle B$ and $\angle C$, $AB = BC = CD = 10$, and $AC + BD = 24\sqrt{2}$. Let M and N be the midpoints of $\overline{AC}$ and $\overline{BD}$, respectively. If $MN = 5$, what is AD^2?

(A) 288 **(B)** 320 **(C)** 384 **(D)** 432 **(E)** 486

> **Remark.** This has got to be the best geometry problem I've ever written. There's a lot of interesting stuff here, and I'm absolutely shocked that it happened after just noodling around. I wish I had released this on a better test, though.

92. **(2023 KMMC 2A P5)**
Aki and Fuyu each have 10 coins. After Aki gives Fuyu 3 of his coins, how many more coins will Fuyu have than Aki?

(A) 3 **(B)** 5 **(C)** 6 **(D)** 8 **(E)** 10

93. **(2023 KMMC 2A P9)**
Karate is doing laundry. In his basket, he has 5 shirts, which each take 9 minutes to fold and 1 minute to put away. In how many minutes can Karate fold and put away all of the shirts?

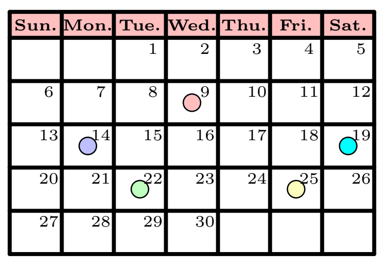
(A) 20 **(B)** 30 **(C)** 40 **(D)** 50 **(E)** 60

> **Remark.** This problem was created with the AoPS users **pandabearcat** and **pog**.

94. **(2023 KMMC 2A P11)**
Tomomi is counting stars. She counts 11 stars in the sky. Later in the night, she counts 8 stars in the sky. If she counted 4 stars twice, how many different stars did Tomomi count?

(A) 11 **(B)** 14 **(C)** 15 **(D)** 19 **(E)** 20

95. **(2023 KMMC 2A P16)**
The calendar below has all the days of November. Karate wants to choose a day in November to hang out with his friend, Judo. He does not want to choose a day that is a Wednesday or a Friday, and he cannot choose a day when he already has an event, all of which are labeled in the calendar with a circle. How many possible days could Karate choose?



(A) 16 **(B)** 18 **(C)** 19 **(D)** 21 **(E)** 25

> **Remark.** Sorry, as my manga classmate says, "I'm just winging it." That was probably a comment from me inserting the circles on random dates. Also, a lot of Asymptote for a pretty simple problem.

96. **(2023 KMMC 2A P18)**
Misa is coloring some pebbles during art class. After Misa colors 8 of the pebbles, Andrew throws away 5 of the pebbles, 2 of which are not colored. If Misa has an equal number of colored and not colored pebbles left, how many pebbles in total were there originally?

(A) 8 **(B)** 10 **(C)** 11 **(D)** 13 **(E)** 15

> **Remark.** Shoutout to two of my friends from the Japanese Conversation Table (UIUC).

97. **(2023 KMMC 2A P22)**
Kenta has 1 walnut, 3 peanut, and 4 pineapple stickers. Ichiro has 2 grape, 1 melon, and some coconut stickers. Ichiro has fewer stickers than Kenta does, but if Kenta gave Ichiro his walnut sticker, then Ichiro would have more stickers than Kenta. How many coconut stickers does Ichiro have?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

> **Remark.** Shoutout to one of my friends from the Japan Student Association (GT).

98. **(sgossk P4)**
When Chirashizushi was 10 years old, Okonomiyaki was 18 years old, and Oyakodon was 30 years old. In how many years will the ages of Chirashizushi, Okonomiyaki, and Oyakodon form a geometric sequence in that order?

> **Remark.** Nice names, past me.

99. **(2023 KMMC 2B P2)**
At a store, Karate bought two chocolate bars, three milkshakes, and four donuts. How many total items did Karate buy?

(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

100. **(2023 KMMC 2B P3)**
How many digits of the number 8935 are odd?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

101. **(2023 KMMC 2B P7)**
Karate is playing a video game with ten levels. If Karate skips two of the first five levels and four of the last five levels, but he beats the other levels, then how many levels does Karate beat?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

102. **(2023 KMMC 2B P9)**
Aki is counting sheep. In the first minute, he counts 17 sheep. In the second minute, he counts 13 sheep. In the third minute, he counts an odd number of sheep, more sheep than in the second minute, and fewer sheep than in the first minute. How many sheep does Aki count in those three minutes?

(A) 41 **(B)** 42 **(C)** 43 **(D)** 44 **(E)** 45

103. **(2023 KMMC 2B P12)**
Ryan has a number. If he subtracts 10 from the number, his result will be 8. If he instead adds 4 to the number, what will his result be?

(A) 6 **(B)** 12 **(C)** 16 **(D)** 22 **(E)** 26

104. **(2023 KMMC 2B P13)**
A 2-digit number has two even digits that have a sum of 12, and the tens digit of the number is greater than the units digit. What is the number?

(A) 48 **(B)** 64 **(C)** 66 **(D)** 84 **(E)** 86

105. **(2023 KMMC 2B P14)**
Karate has 37 strands of pasta, and he cuts 10 of the strands into two smaller strands of pasta. How many strands of pasta does he have now?

(A) 42 **(B)** 45 **(C)** 47 **(D)** 52 **(E)** 57

106. **(2023 KMMC 2B P20)**
How many 3-digit numbers are there such that all its digits are either 0 or 1?

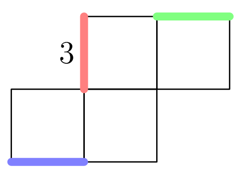
(A) 1 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 8

107. **(2023 KMMC 2B P21)**
Karate last ate at a restaurant on a Monday, and he last ate at a dining hall on a Wednesday, which was 4 days ago. Which of the following could be the number of days since he last ate at a restaurant?

(A) 13 **(B)** 14 **(C)** 15 **(D)** 21 **(E)** 22

Remark. This problem was created with the AoPS user **pog**.

108. **(2023 KMMC 2B P22)**
The picture below shows four squares glued together by their sides. For each square, the numbers 1, 2, 3, and 4 are labeled on its sides in clockwise order. The labels of any two sides glued together are the same, and the label of the red side is 3. What is the sum of the labels of the blue and green sides?



(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

109. **(2023 KMMC 2B P23)**
A number $\square$ is smaller than a number $\triangle$, but the number $\square + 5$ is greater than the number $\triangle + 3$. Which of the following is a possible value of $\triangle - \square$?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

110. **(2023 KMMC 2B P24)**
Ayaka, Judo, and Saya each buy a cake with the same size. Then, Ayaka, Judo, and Saya cut their cakes into 4, 4, and 2 equally-sized pieces, in that order. Finally, Ayaka, Judo, and Saya eat 1, 2, and 1 of their own pieces, in that order. Who eats the greatest amount of cake?

(A) Ayaka **(B)** Judo **(C)** Saya **(D)** Ayaka and Judo
(E) Judo and Saya

Problems (2023)

1. **(2023 KMJJIME I P1)**
Today is Karate's birthday. To celebrate, he decides to buy a cake and 20 candles. If he splits the cake into 6 pieces and puts 3 candles on 5 of the pieces, find the greatest number of candles he can put on the sixth piece.

2. **(2023 KMJJIME I P2)**
Azusa, Ui, and Jun have 120 dollars in total. Jun has as many dollars as Azusa and Ui combined, and Azusa and Ui have the same number of dollars. Find the total number of dollars Ui and Jun have.

> **Remark.** This problem was created with the AoPS users **pandabearcat** and **pog**.

3. **(2023 KMJJIME I P3)**
Mio has two boxes, each with 15 pencils. After Yui, Ritsu, and Mugi each take two pencils from either box, one of the boxes has 11 pencils left. Find the number of pencils the other box has left.

4. **(2023 KMJJIME I P6)**
A certain 2-digit number can be written as the sum of two one-digit numbers, but not as the sum of two *different* one-digit numbers. Find this number.

5. **(2023 KMJJIME I P8)**
Azusa has two different positive whole numbers that multiply to 32. If she adds 21 to the smaller number and adds 12 to the larger number, then the distance between the new numbers on a number line would be 5. Find the sum of all possible sums of the original numbers.

6. **(2023 KMJJIME I P10)**
Ritsu is playing a video game. To beat the game, she must press the A, B, and X buttons twice each. However, if she ever presses the X button twice in a row, then she loses. Find the number of ways for Ritsu to beat the game.

> **Remark.** This problem was created with the AoPS user **PandaMC**.

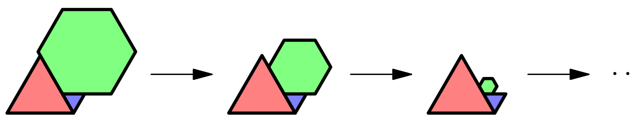
7. **(2023 KMJJIME I P11)**
Karate, Judo, and Aki each choose a positive whole number. If Karate's number is twice Judo's number, Aki's number is four more than Karate's number, and the product of Judo and Aki's numbers is divisible by 21, find the smallest possible sum of Karate, Judo, and Aki's numbers.

> **Remark.** This problem was created with the AoPS user **pog**.

8. **(2023 KMJJIME I P12)**
Mio writes a whole number on a piece of paper. Each minute, she erases the number on the paper, randomly adds either 1, 2, or 3 to the number, doubles the resulting sum, and writes the result. After four minutes, Mio writes the number 780. Find the smallest number Mio could have started with.

> **Remark.** I kind of like this one.

9. **(2023 KMJJIME I P13)**
Mugi has two equilateral triangles with side lengths 6 and 2 that meet at a vertex and a side. On Karate's birthday, she rests a regular hexagon-shaped balloon exactly on the sides of both triangles. After Karate's birthday, the balloon deflates while keeping its shape. At some point, the perimeter of the combined shape is $27\frac{1}{3}$. Find the perimeter of the balloon at that point.



> **Remark.** This problem was created with the AoPS user **pog**. This one took quite a bit of effort to make, and it has a troll in it.

10. **(2023 KMJJIME I P14)**
Yui picks 5 whole numbers each from 1 to 35, inclusive, where any two numbers are at least 4 apart. If three of the numbers Yui picked are 16, 21, and 30, find how many ways Yui could have picked the other two numbers.

11. **(2023 KMJJIME II P1)**
Taiki's birthday was nine weeks ago. To celebrate, he decided to buy 6 cakes for himself and 62 other people. If Taiki cut each cake into the same number of pieces, and everyone was able to have at least one piece, find the smallest number of pieces Taiki could have cut each cake into.

> **Remark.** It actually was nine weeks ago when the test was released.

12. **(2023 KMJJIME II P2)**
At Jakka Jan's Pizza, Yui buys a pepperoni pizza with eight slices, where each slice is supposed to have an equal number of pepperoni pieces. However, one of the slices has one more pepperoni piece than each of the other slices. If the total number of pepperoni pieces is 57, find the number of pepperoni pieces that the slice with the most pepperoni pieces has.

13. **(2023 KMJJIME II P5)**
Daniel adds a two-digit number to the number 78 and gets a two-digit number. Daniel then reverses the digits of his two-digit number, adds it to the number 78, and gets a three-digit number. Find the largest possible value of Daniel's original two-digit number before reversing its digits.

14. **(2023 KMJJIME II P6)**
A group of students are standing in a line, including Saya. At first, there were 29 students in front of Saya and 14 students behind her. After 99 students joined the line, the same number of students were in front of Saya and behind her. Find how many of those 99 students are in front of Saya.

15. **(2023 KMJJIME II P8)**
On his way to school, Aki saw 12 birds. During school, Aki saw 14 birds. On his way back home, Aki saw 11 birds. If Aki saw 7 birds exactly twice and 2 birds all three times, find the number of birds he saw exactly once.

16. **(2023 KMJJIME II P11)**
Aryan, Ryan, and Yan are trying to count to two minutes. They each start counting at the same time and count seconds at a constant rate. After counting, they have the following conversation:

- **Aryan:** Ryan counted to a minute 10 seconds before Yan counted to two minutes.
- **Ryan:** Yan counted to a minute 35 seconds before Aryan counted to two minutes.
- **Yan:** Aryan counted to a minute 15 seconds before Ryan counted to two minutes.

Find the number of seconds it took Ryan to count to two minutes.

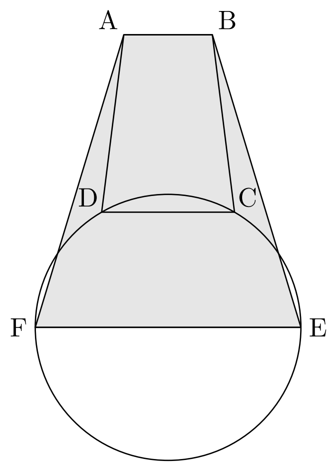
> **Remark.** This problem was created with the AoPS user **pog**. This one is quite hard for a P11.

17. **(2023 KMJJIME II P15)**
In a classroom, each student has exactly two pencils, each of which is either orange, yellow, or gray. If 56 students have an orange pencil, 48 students do not have a gray pencil, and 34 students have two pencils with different colors, find the number of students that have a yellow pencil.

> **Remark.** The fact that there is actually enough information given for the answer to be unique is pretty crazy, don't you think?

Problems (2024)

1. **(2024 WMC Chapter Target P4)**
In the figure shown, AB = 4 cm, CD = 6 cm, and EF = 12 cm. The circle with diameter EF passes through points C and D. If trapezoid ABCD has area 40 cm^2, what is the area of trapezoid ABEF? Express your answer as a decimal to the nearest tenth.



> **Remark.** This problem was originally proposed to the never-released DeMathCounts Year 2 Series. I also did not realize that this problem made it onto WMC until January 5, 2026.

2. **(2024 KMMC 2A P1)**
Judo has 5 rocks. On Monday, Aki gives Judo 7 rocks. On Tuesday, Hana takes away 3 of Judo's rocks. How many rocks does Judo have now?

(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

3. **(2024 KMMC 2A P2)**
How many letters in the word $KARATE$ are consonants?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

4. **(2024 KMMC 2A P3)**
Taiki has a goal to run at least 5 miles a day. If Taiki runs 4 miles on Monday, 3 miles on Tuesday, 7 miles on Wednesday, and 6 miles on Thursday, on how many days did Taiki reach his goal?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

> **Remark.** This problem was created with the AoPS user **pog**.

5. **(2024 KMMC 2A P4)**
Ritsu is writing a book. She has to write at least 80 words, and she has 46 words so far. At least how many more words does Ritsu have to write?

(A) 33 **(B)** 34 **(C)** 35 **(D)** 44 **(E)** 45

6. **(2024 KMMC 2A P5)**
Which of the following values is even?

(A) $4+8$ **(B)** $5-2$ **(C)** $3+4$ **(D)** $7+4$ **(E)** $9-4$

7. **(2024 KMMC 2A P6)**
Yui and Mio are collecting shells. At the start, Yui has 5 more shells than Mio. After Yui gives some of her shells to Mio, Yui now has 3 fewer shells than Mio. How many shells did Yui give to Mio?

(A) 1 **(B)** 4 **(C)** 5 **(D)** 8 **(E)** 10

8. **(2024 KMMC 2A P7)**
What is the smallest two-digit number whose units digit is greater than its tens digit?

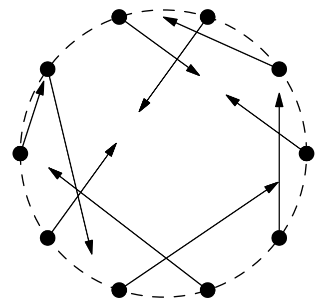
(A) 10 **(B)** 11 **(C)** 12 **(D)** 22 **(E)** 23

9. **(2024 KMMC 2A P10)**
Azusa recorded the temperature in degrees Celsius over 5 days. Each day, the number of degrees is 5 higher than the day before. If she recorded 19 degrees on the first day, how many degrees did she record on the last day?

(A) 34 **(B)** 39 **(C)** 44 **(D)** 49 **(E)** 54

> **Remark.** This problem was created with the AoPS user **pog**.

10. **(2024 KMMC 2A P11)**
How many of the 10 arrows are pointing directly towards each other?



(A) 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 8

> **Remark.** This problem was created with the AoPS user **pog**. Kind of ambiguous wording that somehow got accepted.

11. **(2024 KMMC 2A P12)**
Yui writes a number on a slip of paper. Ui takes Yui's number, subtracts it from 12, and writes that number on the slip of paper. If Ui wrote the number 4, what number did Yui write?

(A) 4 **(B)** 6 **(C)** 8 **(D)** 12 **(E)** 16

12. **(2024 KMMC 2A P21)**
Yui is taking a test. Of the first ten questions, she answers six of them, and of the last ten questions, she answers six of them. If Yui answered eight questions in total, at most how many questions could be on the test in total?

(A) 12 **(B)** 14 **(C)** 16 **(D)** 18 **(E)** 20

13. **(2024 KMMC 2A P23)**
Judo has a drawer with 10 socks, of which there is at least one red, one blue, and one green. He peeks into the drawer and sees seven of the socks – three blue and four red. How many of the following facts must be true?

- There are more red socks than blue socks in the drawer.
- There are fewer green socks than red socks in the drawer.
- At least five of the socks in the drawer are not blue.
- At most five of the socks in the drawer are not red.

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

> **Remark.** This problem was originally proposed to the never-released 2023 DMC 8A.

14. **(2024 KMMC 2B P1)**
How many of the six numbers below are greater than 12?

18 31 12 9 10 24

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

15. **(2024 KMMC 2B P2)**
How many numbers from 11 to 51 have a last digit of 3?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

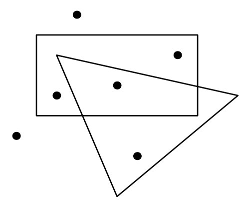
16. **(2024 KMMC 2B P3)**
Which of the following groups of numbers has a sum of 14?

(A) $1, 12$ **(B)** $2, 9$ **(C)** $2, 3, 4$ **(D)** $3, 4, 7$ **(E)** $2, 3, 3, 4$

17. **(2024 KMMC 2B P4)**
Matsuri is thinking of a number. It has two digits, is even, and does not start with 4. Which of the following numbers could Matsuri be thinking of?

(A) 5 **(B)** 13 **(C)** 42 **(D)** 68 **(E)** 124

18. **(2024 KMMC 2B P5)**
How many dots are inside the rectangle?



(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

19. **(2024 KMMC 2B P6)**
Sasazuka has to write a sentence for Japanese class. He has written 26 characters so far and needs to write 18 more before finishing. How many characters long will Sasazuka's sentence be?

(A) 8 **(B)** 18 **(C)** 26 **(D)** 44 **(E)** 52

20. **(2024 KMMC 2B P7)**
Nijika is riding her bicycle from home to school. On her way to school, she went to the cafe to eat for 16 minutes. If Nijika left home at 8:00 AM and arrived at school at 8:43 AM, how many minutes did Nijika ride her bicycle?

(A) 27 **(B)** 37 **(C)** 43 **(D)** 47 **(E)** 59

21. **(2024 KMMC 2B P9)**
Ana has the number 89523. Which of the following numbers can be created by switching exactly two of the digits?

(A) 23598 **(B)** 25389 **(C)** 38592 **(D)** 59823 **(E)** 82395

22. **(2024 KMMC 2B P10)**
The word below has 11 Us, and the word UU does not appear in it.

UGGUGGUGUGGGUGGGGUGUGUGUGGGUGGU

How many times does the word GU appear?

(A) 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

> **Remark.** This problem was created with the AoPS user **pog**. GU!

23. **(2024 KMMC 2B P11)**
Ana and Matsuri are taking turns skip-counting by 7, starting with Ana saying the number 12, then Matsuri saying the number 19, and so on. What is the first number above 50 that Matsuri says?

(A) 54 **(B)** 57 **(C)** 61 **(D)** 65 **(E)** 68

24. **(2024 KMMC 2B P12)**
Every day, Satake eats at least 2 bowls and at most 4 bowls of food. Over five days, Satake ate 15 bowls of food. On at most how many of those days could Satake have eaten 4 bowls of food?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

25. **(2024 KMMC 2B P13)**
Konata and Kagami are racing. Kagami starts 5 meters ahead of Konata. After one hour, Konata was 3 meters ahead of Konata. How many more meters did Konata run than Kagami?

(A) 2 **(B)** 4 **(C)** 6 **(D)** 8 **(E)** 10

26. **(2024 KMMC 2B P14)**
On her way to school, Ana saw five birds. On her way back home, Ana saw eight birds. If Ana saw exactly three birds twice, how many birds did Ana see only once?

(A) 1 **(B)** 4 **(C)** 7 **(D)** 10 **(E)** 13

27. **(2024 KMMC 2B P16)**
How many ways are there to order the numbers 1, 2, 3, 4 so that 2 is not next to 1 or 3?

(A) 2 **(B)** 4 **(C)** 6 **(D)** 12 **(E)** 18

28. **(2024 KMMC 2B P18)**
Miu, Chika, Matsuri, Ana, and Nobue each have a different number from the list 3, 12, 17, 23, and 34.

- Miu does not have the largest number.
- Matsuri's and Ana's numbers are the closest to each other.
- Nobue's number is odd.

Which number does Chika have?

(A) 3 **(B)** 12 **(C)** 17 **(D)** 23 **(E)** 34

> **Remark.** I wrote this problem in like two minutes. Somehow it turned out pretty well for its position.

29. **(2024 KMMC 2B P20)**
Ryan's food stand serves cookies and soda. The table below shows the amount of sugar and price of one cookie and one soda.

Item	Sugar	Price
Cookie	5g	\$4
Soda	7g	\$5

Taiki wants to spend at most \$16 with at least 14 grams of sugar. Which of the following options will Taiki like?

(A) 3 cookies, 1 soda **(B)** 5 cookies **(C)** 1 cookie, 2 sodas
(D) 2 cookies **(E)** 9 sodas

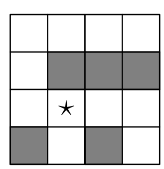
> **Remark.** Shoutout to my Engineering Optimization class for inspiration.

30. **(2024 KMMC 2B P22)**
Miu has four numbers, of which two are equal to $\square$ and the other two are equal to $\triangle$. When she adds three of the numbers, she gets 26, and when she adds all four of the numbers, she gets 38. What is the distance between $\square$ and $\triangle$ on the number line?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

> **Remark.** This problem is somewhat based on one that I wrote for mathleague.org.

31. **(2024 KMMC 2B P24)**
Yui is walking in the grid below. Every move, she walks to one of the **white** squares that share a side with the square she is currently on. If Yui is currently on the square marked with $\star$, on how many different squares could Yui be standing after three moves?



(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

32. **(2024 KMJJIME P2)**
Frusciante and Satake each have 5 golden rings. In one turn, Frusciante can choose to give Satake exactly 2 rings, or Satake can choose to give Frusciante exactly 3 rings. (A person must have enough rings to give rings.) Find the fewest possible number of turns before Satake has 0 rings.

> **Remark.** These names were used in the anime *Ichigo Mashimaro*.

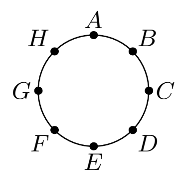
33. **(2024 KMJJIME P3)**
Cindy has a list of five consecutive whole numbers. She chooses three of them, doubles them, adds them together, and gets 70. She adds the other two numbers and gets 25. Find the smallest number in Cindy's original list.

34. **(2024 KMJJIME P7)**
Chika has four different whole numbers. She sees that

- exactly 1 of them is a multiple of 5,
- exactly 2 of them are even, and
- exactly 3 of them are one-digit numbers.

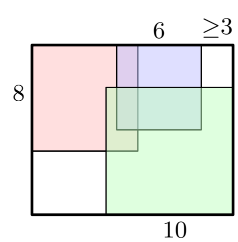
If the sum of the numbers is 18, and the largest number is 9 more than the smallest, find the product of the other two numbers.

35. **(2024 KMJJIME P9)**
Eight equally spaced points lie on the circumference of a circle and are labeled A, B, C, D, E, F, G, and H in clockwise order. Karate draws segments connecting any two different points together. Then, Judo erases every segment that connects two points, both of which are either A, B, E, or G. Find how many segments are not erased.



> **Remark.** This problem was originally proposed to the 2023 KMJJIME II, but it was pushed back due to negative feedback on this problem. We ultimately decided to still use this problem for the 2024 KMJJIME.

36. **(2024 KMJJIME P11)**
Three squares are placed inside a larger rectangle. Taiki wants to extend the sides of the rectangle and shift the squares so that none of the three squares overlap. He wants the blue square to remain at least 3 units away from the right side of the rectangle, and he wants the red square to remain completely to the left of the blue square. Find the least possible perimeter of the rectangle.



> **Remark.** This problem was created with the AoPS user **pog**.

37. **(2024 KMJJIME P13)**
Ana and Matsuri are taking turns removing cards from a stack of 37 cards. Ana goes first, and on each turn, Ana always removes from 3 to 5 cards from the stack, and Matsuri always removes 4 cards from the stack. At some point, after Ana takes her turn, there are exactly 4 cards left. Find the number of different ways that this could happen.

38. **(2025 DMC 8 P1)**
What is the tens digit of
$$20{,}252{,}025 + 20{,}002{,}500 + 2{,}025 + 20 + 25?$$

(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

> **Remark.** This problem was inspired by 2024 AMC 8 P1.

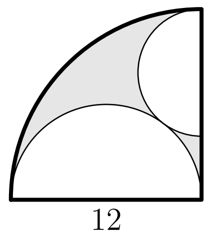
39. **(2025 DMC 8 P9)**
Let the letters D, M, and C represent digits. Billy writes the true equation
$$\underline{D}\,\underline{2}\,\underline{M} \times 3 = \underline{M}\,\underline{7}\,\underline{C}.$$
What is the value of $D + M + C$?

(A) 7 **(B)** 9 **(C)** 12 **(D)** 13 **(E)** 16

40. **(2025 DMC 8 P10)**
When Patrick wrote the sum of the numbers from 2 to 9, inclusive, he accidentally added one of the numbers n times in total, resulting in the number 99. What is the value of n?

(A) 2 **(B)** 5 **(C)** 7 **(D)** 12 **(E)** 13

41. **(2025 DMC 8 P20)**
A semicircle is drawn inside a quarter circle with radius 12 such that its diameter is shared with a radius of the quarter circle, and another semicircle is drawn so that its diameter is on the other radius of the quarter circle, and it touches the other semicircle at exactly one point.



What is the area of the region inside the quarter circle and outside both semicircles?

(A) 9π **(B)** 10π **(C)** 11π **(D)** 12π **(E)** 13π

> **Remark.** The diagram was created by the AoPS user **pog**.

42. **(2025 DMC 8 P22)**
Students Arn, Bob, Cyd, Dan, and Eve are arranged in that order in a circle. They start counting: Arn first, then Bob, and so forth. Arn says the number 121, after which each person says a number either 4 or 6 more than the previous number. At the end, Bob says the number 169. How many times did a student say a number 4 more than the previous number?

(A) 2 **(B)** 6 **(C)** 9 **(D)** 11 **(E)** 16

43. **(2025 DMC 8 P23)**
David has six blocks arranged in the following order.

A B C D E F

Every move, he chooses two of the blocks and switches their positions. After three moves, he ends up with the following arrangement.

C D E B A F

How many possible sequences of three moves are there?

(A) 3 **(B)** 6 **(C)** 8 **(D)** 9 **(E)** 12

> **Remark.** This problem was created with the AoPS user **pog**.